THE ARITHMETIC TEACHER

Volume 8, number 4 **APRIL 1961**

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How arithmetic can increase understanding of science and social studies, how the versatile number runner and the peg board can be used to aid in the teaching of arithmetic, and a report on the Greater Cleveland Mathematics Program.

A JOURNAL OF *The National Council of Teachers of Mathematics*

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All editorial correspondence, including books for review, should be addressed to the Editor. All manuscripts submitted for publication should be in duplicate copy. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

THE NATIONAL COUNCIL
OF TEACHERS OF MATHEMATICS
1201 SIXTEENTH STREET, N.W.
WASHINGTON 6, D.C.

Second-class postage paid at Washington, D.C., and at additional mailing offices.

THE ARITHMETIC TEACHER is published monthly eight times a year, October through May. The individual subscription price of $5.00 ($1.50 to students) includes membership in the Council. (For an additional $3.00, or $1.00 to students, the subscriber may also receive *The Mathematics Teacher.*

Institutional subscription: $7.00 per year for each journal. Single copies: 85¢. Make remittance payable to The National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C. Add 25¢ per year for mailing to Canada, 50¢ per year for mailing to other foreign countries.

As we read

E. W. HAMILTON *Associate Editor*

Problem-solving, the ultimate goal of most education, is not limited to arithmetic nor even to the whole of mathematics. As arithmetic teachers, we profess to deal with the quantitative and spatial aspects of the common problem that man encounters in his effort to adjust to or to control his environment.

If we define the activity of problem-solving, just as John Dewey described the complete act of thought, we start with "a felt need," a state of doubt or indecision as to which of several choices to make in resolving the doubt and restoring a state of equilibrium.

This disequilibrium, described by the term, "felt need," is not the result of innocence or ignorance. We have to be aware of a problem before we can even be undecided about its solution. We also have to be concerned. Unless there is anxiety about the solution, we are not in a state of doubt, and we are not ready to attack a problem.

The first job of the teacher is to bring about this awareness and to enhance situations so that they challenge the student. The second job of the teacher—second in order of position although equal to the first in importance—is to raise the right questions in order to help the student form a plan of attack.

The success of *fundamental problem units* must be attributed, in part at least, to the fact that they draw on common experiences and thus capitalize on already existing awareness. Furthermore, they operate in an area such as household economics where genuine anxiety frequently exists ready-made.

Some of the experimental materials now available from several sources take an entirely different approach. A number of instances have already been reported in which children who used such material performed surprisingly well on conventional standardized tests. This suggests the strong possibility of attaining the usual goal of economic competence as a by-product, so to speak, of working with ideas and symbols in a much more general way.

This does not come about automatically. The creation of awareness and the expansion of that awareness to a magnitude sufficient to capture the imagination of children require a teacher with a different understanding of how numbers work.

The evidence presented by Nelson and Worth as to the superiority of Canadian trainees in education should give us pause. The strong possibility that this superiority is due to a better high school background in mathematics should set us to exploring the backgrounds of our prospective teachers, as well as the rigor of the collegiate courses we offer them. We are still suffering from some of the sins of the recent past and are only gradually emerging from a period of very casual selection of arithmetic teachers, as well as casual attention

to the place of arithmetic in the curriculum. Within the last ten years, I visited a Midwestern city where the established policy was to hire grade-school music teachers whose further assignments were to teach departmentalized arithmetic. This was no worse, I suppose, than the practice of hiring a coach and then assigning him to teach high school mathematics. This was common in my high school days and may still be practiced in the hinterland. At any rate, I submit the above as an example of the ultimate in casualness about the arithmetic program. The fact that any problem-solving ability develops under such circumstances must be attributed to chance, to the natural curiosity of childhood, and to the sagacity of interested parents.

Introspection is a powerful tool. The reader who tests assertions against his own experience as a learner may be hard to please, but he profits from his reading.

With due regard for your own biases, consider the following questions as you read this issue of The Arithmetic Teacher:

1. What effect, if any, will a careful distinction between exercises and problems have on the child? Would it have helped us at the same age?
2. What is the child's reaction to contrived problems? Does he insist upon reality in his stories, games, and other activities?
3. What about all those suggested techniques—the do's and don't's of the textbooks? How many do we remember as being effective in our own childhood?
4. Do you remember any occasion when you were given the opportunity to explain a project?
5. What is the present balance between the demands of business and the demands of technology? This requires a judgment as to usefulness, not in terms of the past fifty years but in anticipation of the next fifty.
6. If, as Brownell suggested years ago, maturity depends upon experience and not merely upon the passage of time, how much and what should we teach in the early grades?
7. What drove us as we learned to solve problems? What intrigued and stimulated us? How might we have reacted to some of the techniques and materials now available?
8. What is our own personal view of the learning process? Is it that of a gradually rising curve, which records small, regular increments, or is it characterized by irregular, often steep and abrupt jumps from one level of learning to another?

These questions are not intended to be either complimentary or critical with respect to the present articles. They are meant merely as a help in promoting critical reading.

Mathematical competence of prospective elementary teachers in Canada and in the United States

L. DOYAL NELSON AND WALTER H. WORTH
University of Alberta, Edmonton, Canada
Both Dr. Worth and Mr. Nelson are faculty members of the Division of Elementary Education at the University of Alberta. Mr. Nelson has particular responsibilities for the mathematics education of elementary teachers in training.

Studies like those conducted by Glennon,[1] Weaver,[2] and Phillips[3] indicate that many prospective elementary teachers in the United States have neither facility in the computational processes which they are expected to teach nor a firm grasp of the mathematical concepts which underlie these processes. Do prospective elementary teachers in Canada exhibit a similar lack of facility and understanding in the field of mathematics? One way to try to answer this question is to compare the mathematics test performance of selected groups of prospective elementary teachers in Canada and the United States. The present article reports on a study in which this was done.

Purpose of the study

The specific purpose of the study was to compare the mathematical facility and understanding of selected groups of prospective elementary teachers in Alberta, Illinois, and Massachusetts.

Method

Subjects

The subjects of the investigation were 468 prospective elementary teachers enrolled in the Faculty of Education at the University of Alberta in September, 1959. Included were 311 one-year Junior Elementary students and 64 first-year, 66 second-year, 21 third-year, and 6 fourth-year students with Bachelor of Education degrees. The mathematics preparation and teaching experiences of these students is indicated in Table 1.

The comparative subjects were 52 seniors in elementary education observed by Phillips[4] at the University of Illinois and 348 sophomores, juniors, and seniors in the elementary teacher-education program examined by Weaver at Boston University.[5]

Instruments

The Phillips Achievement Test in Elementary Arithmetic[6] was used to measure facility in mathematical processes. The Phillips test consists of forty items designed to assess operational skill in performing the conventional operations in whole numbers, common fractions, decimal fractions, per cents, and verbal problems which are commonly taught in the elementary school. Understanding of the mathematical concepts underlying these processes was measured by Glennon's Test of Basic Mathematical Understanding.[7] Glennon's test is made up of eighty items covering five areas of basic mathematical understandings: the decimal system of notation, integers and processes, fractions and processes, decimals and processes, and rationale of computation.

Table 1

Comparison of students enrolled in elementary education in September, 1959, with regard to mathematics preparation and teaching experience expressed in per cent

	Junior Elementary		*Bachelor of Education 1*		*Bachelor of Education 2*		*Bachelor of Education 3*		*Bachelor of Education 4*	
	Yes	*No*	*Yes*	*No*	*Yes*	*No*	*Yes*	*No*	*Yes*	*No*
Grade 12 mathematics ("B" or higher standing)	60	40	94	6	83	17	38	62	50	50
University mathematics (Content)	0	100	0	100	0	100	14	86	17	83
Methods of teaching mathematics	0	100	0	100	41	59	95	5	100	0
Teaching experience	0	100	0	100	20	80	71	29	33	67

Data collection

The tests were administered at the time of registration in September, 1959. No time limits were applied to the administration of either test. The students were permitted to work at their own rate and take as long as they wished to complete the tests. All students took the Phillips test and then the Glennon test. The scoring was done according to a key by a trained marker. Weaver followed a similar procedure in giving the Glennon test to the Boston students during the period from 1953 to 1955. Phillips administered his test to the Illinois group in 1959 in much the same fashion.

Data concerning the students' backgrounds were secured by means of a simple questionnaire, completed by the students at the outset of the testing period and from official university records.

Statistical treatment

Means and standard deviations were calculated for each of the Alberta groups on the Phillips and Glennon tests. Comparisons were then made between the mean scores of the Alberta groups and those reported for similar groups in the United States on the same tests. Weighted t values were obtained by the Cochrane-Cox method[8] to test the significance of differences between means. No assumptions were made about the population variance. Critical values for t were set at the .05 level.

Findings

Mathematical processes

The scores made by each of the Alberta groups on the Phillips Achievement Test in Elementary Arithmetic are indicated in Table 2. Also shown in Table 2 are the scores of prospective elementary teachers at the University of Illinois.

As Table 3 shows, the mean scores of each of the Alberta groups were significantly higher than that of the Illinois group. No significant differences were observed, however, among any of the Alberta groups.

Mathematical concepts

The scores made by each of the Alberta groups on the Glennon Test of Basic Mathematical Understanding are indicated in Table 4. The scores of the elementary-education students at Boston University are also shown in Table 4. Use of a composite score for this latter group was supported by Weaver's[9] report that the means of the freshman, sophomore, and senior groups did not differ significantly among themselves.

Table 2

Scores of prospective elementary teachers on the Phillips Achievement Test in Elementary Arithmetic

	Alberta				*Illinois*
	Junior Elementary	*Bachelor of Education 1*	*Bachelor of Education 2*	*Bachelor of Education 3 & 4*	*Seniors*
Number	311	64	66	27	54
Mean	30.84	30.78	31.56	32.46	27.52
Standard deviation	6.51	6.47	5.72	7.25	8.31

Table 3

Significance of difference between means on the Phillips Achievement Test in Elementary Arithmetic

	Bachelor of Education 1		*Bachelor of Education 2*		*Bachelor of Education 3 & 4*		*Illinois Seniors*	
	$t_{crit.}$	$t_{obs.}$	$t_{crit.}$	$t_{obs.}$	$t_{crit.}$	$t_{obs.}$	$t_{crit.}$	$t_{obs.}$
Junior Elementary	1.99	0.07	1.99	0.91	2.05	1.13	2.01	**2.74***
Bachelor of Education 1			2.00	0.73	2.04	1.04	2.01	**2.31***
Bachelor of Education 2					2.04	0.58	2.01	**2.41***
Bachelor of Education 3 & 4							2.04	**2.73***

* Significant beyond the .05 level

As is shown in Table 5, the mean scores of each of the Alberta groups were significantly higher than that of the Boston group. The mean score of the Bachelor of Education 2 group was significantly higher than that of either the Bachelor of Education 1 or Junior Elementary group. Similarly, the mean score of the Bachelor of Education 3 and 4 group was significantly higher than that of either the Bachelor of Education 1 group or the Junior Elementary group.

Implications

From these data it might be inferred that the mathematical competence of prospective elementary teachers in Alberta is relatively higher than that evidenced by their counterparts in Illinois and Massachusetts. Additional support for this hypothesis may be found in the fact that experienced elementary teachers on the Atlantic seaboard[10] and in the state of Utah[11] are reported to have obtained mean scores on the Glennon test of only 43.82 and

Table 4

Scores of prospective elementary teachers on the Glennon Test of Basic Mathematical Understanding

					Boston
	Junior Elementary	*Bachelor of Education 1*	*Bachelor of Education 2*	*Bachelor of Education 3 & 4*	*Composite group*
Number	302	63	66	27	348
Mean	56.17	57.24	60.41	62.11	44.60
Standard deviation	9.27	9.34	7.91	7.79	11.87

Table 5
Significance of difference between means on the Glennon Test of Basic Mathematical Understanding

	Bachelor of Education 1		*Bachelor of Education 2*		*Bachelor of Education 3 & 4*		*Boston group*	
	$t_{crit.}$	$t_{obs.}$	$t_{crit.}$	$t_{obs.}$	$t_{crit.}$	$t_{obs.}$	$t_{crit.}$	$t_{obs.}$
Junior Elementary	1.99	0.82	1.99	3.69*	2.00	3.74*	1.96	14.23*
Bachelor of Education 1			2.00	2.07*	2.03	2.55*	1.99	9.50*
Bachelor of Education 2					2.04	0.95	1.99	13.75*
Bachelor of Education 3 & 4							2.04	10.81*

* Significant beyond the .05 level

52.46, respectively. A further look at Table 4 will show that these scores are well below those obtained by the Alberta groups.

At the same time, however, cognizance should be taken of at least two factors which may have unduly influenced the results of this experiment. The first of these is the time factor. The Boston University groups were tested some five to seven years ago. It may be that tests administered in 1959 would have yielded considerably different scores. The second factor to be taken into account is the difference which may exist in the high school mathematics preparation of Canadian and American students. The vast majority of the Alberta students had completed three years of high school mathematics culminating in a ten-month course in Grade 12 in which the following illustrative topics are included: quadratic functions and equations, series of numbers, polynomials and algebraic equations, permutations and combinations, mathematical induction, and the binomial theorem.

Nevertheless, the findings of this exploratory study do indicate some need for reappraisal of teacher education at the elementary level. For, in general, those groups of students with the most experience in mathematics, in the form of course(s) in mathematics methods or content, evidenced a higher level of performance on the Phillips and Glennon tests. This fact, when coupled with the tendency toward greater variability in test performance among those groups with the least experience in mathematics, seems to suggest what may be the obvious—that competence in mathematics varies directly with the amount of experience in mathematics.

That the obvious can be overlooked, however, is indicated by the persistence of a large number of elementary teacher-education programs in both Canada and the United States which devote less time to the study of mathematics than to any other subject commonly taught in the elementary school. And if the quality of mathematics instruction and hence the level of pupil achievement depend, at least in part, upon the mathematical competence of the teacher, then those who are concerned about the relative mathematical attainments of students in the United States and elsewhere might find the solution to this problem lies in improving preparation programs in mathematics for prospective elementary teachers.

Notes

1. V. J. Glennon, "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic," *The Mathematics Teacher*, XLII (December, 1949), 389-96.

2. W. F. Weaver, "A Crucial Problem in the Preparation of Elementary School Teachers," *Elementary School Journal*, LVI (February, 1956), 255-61.

3. C. A. Phillips, "The Relationship between Achievement in Elementary Arithmetic and Vocabulary Knowledge of Elementary Mathematics as Possessed by Prospective Elementary Teachers," unpublished doctoral dissertation, University of Illinois, 1959.

4. *Ibid.*

5. Weaver, *loc. cit.*

6. Phillips, *loc. cit.*

7. Glennon, *loc. cit.*

8. P. O. Johnson, *Statistical Methods in Research* (New York: Prentice-Hall, Inc., 1949), p. 75.

9. Weaver, *loc. cit.*

10. Glennon, *loc. cit.*

11. J. E. Bean," Arithmetical Understandings of Elementary School Teachers," *Elementary School Journal*, LIX (May, 1959), 447–50.

To the editor:

Two articles in the January, 1961, issue of THE ARITHMETIC TEACHER drew thought-provoking responses from readers. We should like to share the contents of two of these letters with you at this time.

Not until I read Wayne Peterson's "Case in Point" in the January, 1961 issue of THE ARITHMETIC TEACHER did I realize that my comment in an article titled "Number, Numeral and Operation" in the May, 1960 issue could be interpreted as an overstatement. I wish the readers to know that I was referring to the following statement on page 4 of the Appendices on the Report of the Commission on Mathematics.

"Some writers lay considerable stress on the distinction between a number and a name for a number, which they call numeral. This distinction should be appreciated by teachers of high school mathematics. However, too much insistence on the distinction in writing or in speaking may lead to pedantic circumlocutions. In the interest of facile communication, we frequently allow ourselves an elliptical liberty and use the word "number" to refer either to a number or to a name for it."

I believe that my interpretation of the statement by the SMSG to be accurate. I was referring to paragraph 2 on page 17 of Mathematics for Junior High Schools, Commentary for Teachers, Volume I (Part I) of the School Mathematics Study Group which reads as follows:

"A numeral is a written symbol. A number is a concept. Later in the text it may be cumbersome to the point of annoyance to speak of 'adding the numbers represented by the numerals written below.' In such case the expression may be elided (*sic*) to 'adding the numbers below'."

JOHN R. CLARK
Consultant in Mathematics Education
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An article, such as "Percentage—noun or adjective?" by Professor Rappaport has been long overdue. Too many persons, mathematicians included, have been ignorant of the correct usage of the terms "per cent" and "percentage." Congratulations for publishing this article—one which may seem rather trivial to some but not to this reader. Congratulations also to Professor Rappaport for taking the time to inform others on this matter. Let us hope that most readers will consider his pleas carefully and attempt to speak and write correctly with regard to these terms if they do not do so now.

The day I received my January issue of THE ARITHMETIC TEACHER I noted the misuse of the term "percentage" in four places on the front pages of three local papers. One of these was the college weekly. It is not at all infrequent that one reads dissertations in which "percentage" is used as a term when "per cent" is what is meant. A recent published report of a New York state commission studying higher education and the State University repeatedly used "percentage" to head columns of "per cents" in the tables of the report.

It is too bad that all the major news media in the country could not be made aware of this excellent article with the hopes that they would be more careful in the use of these two terms.

Keep up the excellent work in publishing this magazine. Each month it provides material for very stimulating discussions in classes I instruct on the subject of arithmetic.

RUDOLPH J. CHERKAUER,
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Those problem-solving perplexities

CLEATA B. THORPE *Huron College, Huron, South Dakota*
As professor of education at Huron College, Miss Thorpe not only teaches courses in education, including a course in the teaching of arithmetic, but she also supervises student teaching in the elementary schools.

We teachers are in large measure responsible for problem-solving being the obstacle that it is to many a pupil in elementary schools. In the first place, we toss the terms "problem" and "problem-solving" about quite indiscriminately. We seem to have no clear and definite concepts for those terms in our own minds.

Teachers' concepts are vague

In a situation typical of many classrooms, a teacher, Miss A, says, "Work the ten problems on page 21 in your book and then do those I have put on the chalkboard." The ten "problems" in the book are of the nature of $31\times24=$——, $60\times49=$——, and so on, and those on the chalkboard are of the same type.

In another classroom, a teacher, Miss B, says, "Now work the rest of the problems on page 36." These are a set of a type such as (a) "The cookies we want for our party come 12 in a package; how many will we get in 3 packages?" and (b) "If each club member puts in 15 cents, how much money will we have from our 12 members?" All of them require the same type of multiplication equation.

In still another classroom, the teacher, Miss C, sets her class to work saying, "Let's see how many of you can get correct solutions to the four problems at the top of page 42 in the time we have left." The first of these problems is "Ron's father has asked him to help solve a real family problem. His father wants to build a 12′ by 12′ outdoor playpen for one-and-a-half-year-old Perry. His first problem is how many posts will be required if he sets them 6 feet apart?"

Though all these teachers have referred to "problems," the question before us is which, if any, of the three assignment samples are problems? Until we recognize that the computation required, a pupil's past experience, and present environmental circumstances, as well as the mathematical statement at hand, are all involved in determining to what extent a quantitative question is a problem, we have no adequate answer to that question.

There are those who hold to a somewhat categorized classification who would say that Miss A's type of problem is merely a computational exercise, a numerical exercise, with no problem situation involved. These persons would define a problem as a verbal statement involving numbers, in which a question is raised and the arithmetical operation required for obtaining an answer to it is not indicated. In that case, Miss B's type would qualify as problems as would also Miss C's.

There are others who put all arithmetic work into two classifications—computational exercises (often called examples) and verbal problems. For them, Miss A's type is an exercise or an example while Miss B's and Miss C's types are verbal problems.

Then there are still others, whose concepts are less vague, who do recognize the part that pupils' past experience and present environmental circumstances play in identifying a true problem-solving situation. Miss C probably is in this group.

The illustrations above give ample evidence that our arithmetic vocabulary relative to the terms "problem" and "problem-solving" have such vague and indefinite interpretations as to make real understanding difficult. The result is that countless elementary-school pupils, and many college students as well, say, in effect, "I can do the computations, but my trouble is the problems." If teachers at every grade level will clarify their own concepts and use a more precise and accurate vocabulary, the perplexities that accompany problem-solving in arithmetic can be considerably reduced.

Proposals for clarifying concepts

Only when their own concepts in the problem-solving areas of arithmetic have become definite and specific have teachers acquired the insights needed to direct pupil learning with the desired results. Basic to clarifying these concepts is the acceptance of some general understandings. Let us, then, establish as appropriate frames of reference the following proposals.

1. A true problem is taken to be a situation facing an individual or a group who want to find a solution and have no ready-made method for arriving at one. A problem in arithmetic, then, may be said to be a situation described in words involving a quantitative question to which an answer is sought and for which the computational process is not indicated. However, as stated in the previous section, the types of computation required and the pupils' past experience and present environmental circumstances also help determine whether a quantitative question indicates a problem situation.
2. Structuring arithmetic concepts is an essential element of problem-solving, fundamental to both meaning and understanding. The structuring process should begin on a physical level. A child sees 2 blocks and 3 blocks put together into a group of 5 blocks. Next he learns the use of word and number symbols to tell what took place—2 blocks and 3 blocks are 5 blocks. As a next step he dispenses with the physical objects and in imagination aided by memory, he combines small groups and writes the arithmetic sentence—$2+3=5$. Now he has experienced the physical aspects of structuring and their representation by symbols, knowing when he began, however, that the activity was a combining of groups. In other words, the structure had been provided for him and he was merely representing it by use of objects and by symbols. So far he has not really solved problems.
3. A true problem situation may be said to be an unstructured situation. The first step in problem-solving is to structure the situation presented. To do this, a pupil imagines the action taking place or uses a diagram by which to visualize the situation. He then decides upon an equation which will give the indicated relationship to the numerical information provided. His first equation may be a so-called "expanded equation" (actually a generalization) composed of words only; for example, three times distance traveled in one hour equals distance traveled in three hours. He then substitutes the available numerals; for example, $3\times \underline{\qquad} = 165$. Now that he has structured the problem arithmetically, he has practically solved it, for he has previously learned to solve multiplication equations almost automatically and with computational accuracy. Insight assists him in structuring, but it also results from structuring experience in problem situations.
4. Assisting pupils to make a transition from physically structured (experiential) to abstractly structured (mathematical) concepts is a fundamental teaching problem. Only when pupils become proficient in that transitional

ability will they become competent in problem-solving.

Procedures for implementing the proposals

Referring again to Miss A's type of problem; e.g., $31 \times 24 = ?$, there may be pupils in her class who do not know how to obtain the answer called for in the question indicated by the arithmetic symbols. Let us be consistent, then, and not call that a true problem situation when it is simply an inability to handle the mechanical operations indicated by the equation. The structuring has already been done, and there is no problem situation to be analyzed. This type of material we shall classify as *computational exercises.*

Next we may look at an illustration of Miss B's type. If pupils are being introduced to the concept of multiplication as a combining of a number of groups of equal size, having had a good deal of concrete structuring as a first step, and are now putting some of these experiential structures into arithmetical statements (equations), they are solving problems. In the case of illustration (a), "The cookies we want for our party come 12 in a package; how many will we get in 3 packages?" a pupil's problem is to structure this statement and its accompanying question into an appropriate arithmetical equation ready for solution. If he has been well taught up to this point, and now comprehends the generalization that the number of groups times the number in each group gives the total number, he will recognize that he has two of the three elements needed for structuring a simple multiplication equation. Here is where insight plays an important part. Can this pupil translate his perceptual concept (imaginary) into an abstract symbolic concept (an equation)? If he can, insight has guided him, and he has solved a problem. Solution of the already familiar multiplication equation is practically automatic, once the equation has been formulated.

However, after a pupil has acquired the concept and the necessary insight which enables him to recognize the situation and set down the equation readily, he may do many more such computations in finding the answers to other verbal question situations of the same type. These will be "keeping-in-practice" types of lessons, but they are not problem-solving experiences after the structuring has become almost automatic. Therefore, the pupils' past experience and the present environmental situation determine whether Miss B's illustrations (a) and (b) are problems or whether they are merely "keeping-in-practice" exercises. As a matter of fact, however, to some pupils in Miss B's class (a) and (b) may still constitute problem-solving situations, while to others they are no longer problems but are used as exercises for keeping in practice. In such a case, it is Miss B's responsibility to recognize which pupils must still be assisted in problem-solving learning and which have reached the keeping-in-practice purpose for that particular type of lesson material.

In the case of Miss C's classroom, her instructions to the class indicate that here are some *problems*, and that both she and her pupils are familiar with problem-solving procedures. We may assume that her pupils comprehend what is involved in a problem situation—that here is a quantitative statement with a question asked and the numerical operations not indicated. They recognize that their first step is to read the statement of information and the question carefully and to establish some relationship between the numerical information set forth and the numerical answer they are expected to find. For their second step, they must mentally structure (visualize) the physical situation involved—whether it is a combining of groups, a separation of some total amount into its parts, or some other type of procedure. They must also determine whether more than one step is involved or whether one equation is sufficient for solution. They may employ

some visual semiconcrete aids in the form of diagrams. When they have accomplished the physical structuring, either diagramatically or in imagination, they are ready to translate the acquired concept into the abstract, symbolic arithmetical equation or series of equations which indicate the needed operations. After performing those operations and obtaining a tentative answer, their accustomed next step is to go over the problem statement again to verify the reasonableness of the attained answer and perhaps to supply proof that it is correct.

In solving the illustrative problem in Miss C's classroom, then, the pupils' first step will be a careful reading; their second, a ready recognition of how a diagram will help; and their third, a solution arrived at simply by counting the posts indicated on the diagram. If Miss C's pupils can proceed with their assignments in this manner, they will have resolved many of the perplexities of problem-solving.

Better results can be attained

There is no lack of evidence, stemming from research as well as from opinion, that problem-solving has been a weak spot in arithmetic-learning in this country. Much time and energy have gone into studies of the reasons for this prevalent weakness among elementary-school pupils as well as among high school and college students and even adults, without making any marked progress toward improvement of the situation.

In the opinion of the writer, the immediate future holds more promise for dissipating this problem-solving weakness than past decades have provided. If teachers will, as pointed out in earlier paragraphs, clarify their own concepts with some precision of vocabulary and procedure to identify some lesson material as computational exercises, some as keeping-in-practice material, and some as true problem-solving material, much will be accomplished. Once such concepts are clear in a teacher's mind, she will be able to help pupils acquire those concepts and recognize that arithmetic lessons differ in purpose and, therefore, differ in attack procedures. Pupils so taught will recognize such lesson material as Miss C's illustration as the type of situation in which time will be saved by making a careful survey of the problem situation to see what operations are indicated, rather than proceeding at once to manipulate figures in an attempt at solution.

If teachers, in addition to clarifying their own and their pupils' concepts of true problem situations, will apply some of the newer procedural techniques to arithmetic, more progress will be made.

Experimental teaching procedures have decidedly strengthened the theory that understanding lends permanence to acquired skills and to the acquisition of learning in general. The drill method of the past was too often devoid of understanding. There is ample research evidence to show that manipulative experiences, along with exploratory reasoning, the discovery of number relationships, and the evolving of generalizations aid understanding and help insure permanent learning. These experiences are involved in what we have termed structuring, and together characterize what is currently referred to as *reflective teaching*. The use of reflective teaching in problem-solving lessons in arithmetic is most effective when certain conditions prevail.

1. The atmosphere of the classroom must be conducive to pupils feeling accepted and at ease.
2. The attitude of the class must be cooperative and receptive to the ideas and opinions of others.
3. Suitable problem material must be provided.
4. There must be time for unhurried thought and relaxed activity.
5. Teachers and pupils remember that concepts do not come as single flashes of insight; they unfold slowly, step by step, even with guidance.

6. There is recognition of the fact that there is usually more than one correct and acceptable solution to every problem, though some may be too "roundabout" for practicality.
7. Any correct solution is commended.

The values of such learning processes in the teaching of problem-solving in arithmetic have come to be more widely recognized than in the past, and there is every indication that they will play an increasingly important part in elementary arithmetic-learning in the immediate future. By way of summary, we may conclude that there are some very promising possibilities for vanquishing the problem-solving perplexities—if teachers will do their part.

Notes

For further discussion of structuring arithmetic, see Chapter 3, "Structuring Arithmetic," by Henry Van Engen and E. Glenadine Gibb, in *Instruction in Arithmetic*, Twenty-fifth Yearbook of The National Council of Teachers of Mathematics (Washington, D.C.: The National Council, 1960).

For further discussion of learning to structure equations, see Cleata B. Thorpe, "The Equation: Neglected Ally of Arithmetic Processes," *Elementary School Journal*, March, 1960, pp. 320–324.

For further discussion of structuring multistep problems, see Maurice L. Hartung, Henry Van Engen, Lois Knowles, and E. Glenadine Gibb, *Charting the Course for Arithmetic* (Chicago: Scott, Foresman and Company, 1960), pp. 118–119.

For further discussion of learning to discover and generalize, see Chapter 4, "Guiding the Learner to Discover and Generalize," by John R. Clark, in *Instruction in Arithmetic*, Twenty-fifth Yearbook of The National Council of Teachers of Mathematics (Washington, D.C.: The National Council, 1960).

Announcing a new publication for the puzzle buff

Anyone who likes puzzles will want to examine the new *Recreational Mathematics Magazine*, published and edited by Joseph S. Madachy, Box 1876, Idaho Falls, Idaho.

The first issue, dated February 1961, contains material ranging all the way from famous bridge hands and chess problems to word games and magic squares.

A partial list of the contents shows that other topics include Boolean algebra, strategy problems, conics by paper folding, progressions, alpha metrics, primes and factors, color problems, and Mobius strips.

The inclusion of a section called "Readers Research Department," in which readers can pose and discuss unsolved problems, promises to be a forum for good amateurs.

The first issue contains some typographical errors that have been corrected after printing.

Most of the material is not for a child—unless he is a prodigy—but is not really beyond the grasp of the able adolescent and the interested adult.

An approach to problem-solving

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Arithmetic is universally accepted as one of the fundamentals which must be taught to American school children. Yet certain aspects of the teaching of arithmetic have given teachers a great deal of concern. Petty found that, "Complaints concerning the child's lack of ability to solve problems dealing with quantitative situations come from many quarters."[1] Clark and Eads indicated that the difficulty children have with problem-solving lies in their lack of ability to see the various relationships involved in the problem situation.[2] In *What Does Research Say About Arithmetic?* Glennon and Hunnicutt suggest that inability to estimate reasonable answers is a difficulty.[3]

Research findings on problem-solving

Many of the summaries of research relative to problem-solving in arithmetic indicate a need for studies dealing with the role of reading ability. In *The Encyclopedia of Educational Research,* Wilson states, "The question of the nature of the reading instructions that should be given has received only limited attention, and further research is needed before any conclusions can be stated."[4] Corle says, "Understanding the terms used in arithmetic is a definite factor in problem-solving efficiency. Teachers must insure familiarity with the vocabulary of verbal problems if effective problem solving is to result."[5]

In a discussion of the nature of problem-solving in arithmetic, Johnson pointed out that vocabulary is one of the main factors in a student's ability to solve problems.[6] As an example, a student can hardly be expected to solve a problem dealing with the volume of a rectangular solid unless he knows the meanings of volume and rectangular solid, and unless he knows a process for solving the problem.

Despite the fact that many types of studies dealing with problem-solving have been conducted, there is still need for studies in this area. For example, studies are needed which relate to the way children learn arithmetic concepts, the thought processes involved in solving problems, and the manner in which children make choices regarding the processes used. Stevenson has stated that more detailed research should be undertaken to show how much time and effort should be expended in giving pupils a method of attacking problems, in training them to estimate answers, and in assisting them to understand technical words and other phases of instruction.[7] Spitzer and Flournoy state that students of arithmetic teaching need to make studies to determine whether proposed problem-solving improvement procedures found in children's textbooks actually contribute to a student's ability.[8]

Monroe and Engelhart report a study by Stevenson in 1924 which stressed certain aspects of problem-solving similar to

those of this study. For a period of twelve weeks, Stevenson used 1,027 fifth-, sixth-, and seventh-grade pupils who were taught to read and analyze problems and to estimate answers in round numbers. Part of the twelve weeks was used to have pupils state the problems in their own words, to solve problems without the use of numbers, to work a variety of problems dealing with real-life situations, and to estimate answers in round numbers.[9] About Stevenson's study Monroe and Engelhart say, "The gains in achievement are certainly significant. . . . It is unfortunate that control groups were not used."[10]

A new study is undertaken

The problem

The problem in this study was to determine the effect of a particular method on achievement in problem-solving in sixth-grade arithmetic. This method, consisting of a systematic approach to the solution of arithmetic problems, was used consistently with the experimental group. Vocabulary study was emphasized at the beginning of each class period when problems were to be solved. Word meanings, syllabication of words, synonyms, and crossword puzzles were variations of the vocabulary exercises. Following vocabulary study, emphasis was placed on talking or thinking through the total situation which the problem presented. A third phase of the method was emphasis on drawing a simple diagram to indicate that the student understood the problem. This phase was also stressed to help the student decide what arithmetic process should be used to solve the problem. Estimating a reasonable answer was the fourth phase of the method. Rounding off simple numbers, taught early in the year, was intended to prepare the student for estimating answers to more difficult problems later. The final phase of the method required each student to use paper and pencil to find the exact answer to the problems.

Teachers in the control group were instructed to adhere closely to the directions found in the Teacher's Guide of the *Making Sure of Arithmetic* series, 1952 edition. These teachers were also allowed to use specific techniques which had proved successful for them. Because these varied from teacher to teacher, a log was kept so that these variations could be compared with the instructions of the Teacher's Guide and with the procedure used by the experimental group. Table 1 shows a tabulation of these techniques. Those techniques which occurred most often were labeling correctly, checking answers, making original problems, selecting key words, and working at the chalkboard. Techniques suggested by Morton appear in Table 2.

Control of various factors

To insure that the procedure used would be the only variable in the study, certain precautions were taken. Controls were placed on factors such as (1) the length of time for each exercise; (2) homework assigned; (3) instructions regarding the teaching of concepts which occurred in the textbook between pages of problems; (4) attention given to the slow learners; and (5) participation by all children in each problem exercise.

Teacher and pupil pairing

Twenty-two teachers, eleven serving as the control group and eleven serving as the experimental group, were used. As closely as possible, teachers were paired according to the types of degrees held, years of experience teaching sixth-grade arithmetic, and total years of teaching experience. Eleven parishes were used with an experimental class and control class in each parish.

To select subjects for the study, 696 students were screened. Of this number, 168 students were eventually paired according to sex, age, intelligence quotient, and arithmetic reasoning achievement. However, due to the fact that ten of the paired students did not take the second

Table 1

Summary and frequency of techniques used by several teachers of control group for solving problems found in 21 pages of Morton's "Making Sure of Arithmetic," sixth-grade text

Techniques	*Frequency*
Correct labeling	21
Checking answers	13
Selecting correct process to use	7
Making original problems	6
Selecting key words for process to use	5
Reading problems carefully	5
Asking the question another way	3
Practicing neatness	2
Working at the chalkboard	2
Looking for hidden questions	2
Following directions	1
Analyzing "What does the question ask?"	1
Showing relationships of addition to multiplication	1
Doing individual work	1

achievement test, only 74 pairs were used in presenting the results of this study.

Supervision of the study

Supervision of the study was done by local supervisors of instruction and by the state supervisor of elementary education. Local supervisors in the eleven parishes gave needed assistance to teachers and made informal reports of progress to the state elementary supervisor. For the state elementary supervisor, supervision was limited to an orientation session with the supervisors and a similar session with the teachers before the initiation of the study. A visit of one hour was also made to all the classes participating in the study.

Table 2

Summary and frequency of techniques suggested by Morton for teaching 21 pages of problems from sixth-grade text

Techniques	*Frequency*
Reading problems carefully	11
Selecting correct process to use	6
Making practical application of problems	3
Estimating the answer	3
Reading problems orally	2
Finding out what the question asks	2
Seeing the whole idea of the problem	2
Analyzing relationships between quantities	2
Looking for a hidden question	2
Using one answer to solve another problem	2
Rereading problems carefully	1
Checking the answer	1
Making practical application of fractions	1

Presentation of findings

Results of the study were based on the California Arithmetic Reasoning Achievement Tests which were administered in September, 1959, and January, 1960. Scores made by the 74 pairs of students on the second achievement test were used to determine gains or losses in achievement made during the four-month interval. The students in the upper third of the control group had a mean gain of .5 years, while the experimental group had a mean gain of .7 of a year. For the middle third, the mean gain of the control group was .6 of a year compared to a mean gain of .9 years for the experimental group. In the lower third, the mean gain of the control

Table 3

Comparison of mean gains of upper, middle, and lower thirds, as well as total group of experimental and control groups based on California Arithmetic Reasoning Test given September 1959 and January 1960

Group	*Mean gain of control group*	*Mean gain of experimental group*	*Difference of mean gain*	*Critical ratio*	*Level of significance*
Upper third	.5	.7	.2	.9	N.S.*
Middle third	.6	.9	.3	1.66	N.S.
Lower third	.7	.8	.1	.43	N.S.
Total group	.6	.8	.2	1.83	.10

* Not significant at the .10 level

group was .7 of a year, while the experimental group showed a mean gain of .8 of a year. When all 74 pairs of students were grouped together, the mean gain of the control group was .6 of a year and for the experimental group it was .8 of a year.

Tests of significance as shown in Table 3 were applied to the results of the three groups of pairs and to the 74 pairs used in the entire study to determine whether the differences in gain in favor of the experimental group were statistically reliable. The results were not significant at the .10 level when the tests were applied to each small group separately. When applied to all 74 pairs the results were significant at the .10 level.

Conclusions

The conclusions which follow seem justified on the basis of the data gathered during this study.

1. The method used with the experimental group was effective. The total experimental group and each of the subgroups made gains of .7 or more years in the four months of the study.
2. The method used with the control group was also effective. The total control group and each of the three subgroups made gains of .5 or more years in the four months of the study.
3. The method used with the experimental group seemed slightly superior to that used with the control group. For the total sample, the differences in mean gains favoring the experimental group were significant at the .10 level. For the three subgroups, the differences in mean gain slightly favored the experimental group in each case. However, the differences in mean gains were not statistically significant.
4. Further experimentation with a larger sample and a longer period of time is needed to determine whether the experimental procedures used in this study are consistently superior to procedures used with the control group.
5. Further experimentation is needed in the area of learning problem-solving.

Notes

1. Olan Petty, "Requiring Proof of Understanding," THE ARITHMETIC TEACHER, II (November, 1955), pp. 121–123.

2. John Clark and Laura K. Eads, *Guiding Arithmetic Learning* (Yonkers-on-Hudson: World Book Company, 1954), p. 259.

3. Vincent J. Glennon and C. W. Hunnicutt, *What Does Research Say About Arithmetic?* A Report Prepared for the Association for Supervision and Curriculum Development (Washington D.C.: National Education Association, 1958), p. 40.

4. Guy Wilson, "Arithmetic," *Encyclopedia of Educational Research* (New York: The Macmillan Company, 1950), p. 54.

5. Clyde G. Corle, "Thought Processes in Grade Six Problems," THE ARITHMETIC TEACHER, V. (October, 1958), pp. 193–203.

6. J. T. Johnson, "On the Nature of Problem Solving in Arithmetic," *Journal of Educational Research*, XLIII (October, 1949), pp. 110–115.

7. P. R. Stevenson, "Increasing the Ability to Solve Arithmetic Problems," *Educational Research Bulletin*, III (October, 1924), pp. 267–270.

8. Herbert F. Spitzer and Frances Flournoy, "Developing Facility in Solving Verbal Problems," THE ARITHMETIC TEACHER, III (November, 1956) pp. 177–182.

9. Walter S. Monroe and Max D. Engelhart, *A Critical Summary of Research Relating to the Teaching of Arithmetic*, University of Illinois, Bureau of Educational Research, Bulletin 58 (Urbana: University of Illinois, 1931), p. 115.

10. *Ibid.*, p. 115.

Children are naturals at solving word problems

They love to solve puzzles. They love to explore.

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Children have naturally inquisitive minds. They have the ability to shut out all other things in concentrating on the solving of a puzzle. They joyously answer a challenge. All this makes them naturals in solving word problems.

Why is it, then, that many children are fearful of word problems? Certainly we are all familiar with reasons given. Whatever the cause, let's face the problem squarely and see how we can help children with word problems.

Let's begin tomorrow morning by grouping the pupils with no more than twelve or thirteen pupils in a group, the selection based on nothing more than the degrees of freedom from fear in solving word problems. We can explain to the children that we are going to work in groups, that each group will have its turn, and that with fewer of us in a group we will have more opportunity to participate.

Be sure the problems are based on a common experience taken from the lives of one or more of the pupils. Perhaps they went to the grocery store for their parents the previous day.

Write the statement on the chalkboard

Mike bought two loaves of bread at 27¢ a loaf and a gallon of milk at 45¢ a half-gallon. He bought two pounds of oranges for 36¢ a pound.

The children will read the statement silently. Notice that no question is asked. Thus attention is directed to a given situation with the *children* deciding what is to be found.

Dramatize the problem

Ask for volunteers to play the part of the boy and the grocery clerk. Any objects, such as erasers, pencils, or pieces of chalk, can be used to represent the grocery items. If children are hesitant to dramatize the work, you may ask, "I'll be Mike. Who will go with me? Good! Shall we pretend that Mary is the grocery clerk?"

Draw a simple picture of the statement

Ask for volunteers, each of whom will make a simple drawing of all the items to be purchased. It might be wise to set the pace by quickly drawing a circle to show the bag of oranges, thus helping the child to realize that the drawing need not be elaborate. With at least two children, each making a simple drawing at the chalkboard, fear on the part of one child is alleviated and at the same time more children participate. The purpose of both the dramatization and the drawing of a simple picture is to unfold a real understanding of the situation.

Estimate the answer

While the children at the chalkboard are illustrating the problem, direct the atten-

tion of the other children in the group to estimating the answer. They will need help at first: "How much did we pay for one loaf of bread? Isn't 27¢ close to 30¢? How much are two 30's? Can you remember 60¢?"

"Now we want to buy milk. How much milk do we want? How many one-half gallons must we buy? How much will the milk cost?"

"About how much do the bread and milk cost? Can you remember it?"

"Let's estimate the cost of the oranges. Is 36¢ closer to 30 or 40? About how much are two 40's?"

"About how much were we spending for bread? (60¢) What did we estimate for the milk? ($1.00) How much do these two items total? ($1.60) What did our oranges cost? (80¢) About how much will we spend in all?" ($2.50) Write the estimation on the chalkboard.

Examine the drawings on the board

It is wise to say from the very first day, "Henry and Mary were very nice to do the drawings for us, and it is our job to help in any way we can. *We all work together.*" Children rarely write the price of the article by their drawing. If no child suggests it, you might be able to lead them to this later by a remark, such as "How much are we paying for this?" (Point to one of the items in the picture.)

Only now do the children really understand the problem. They have made the problem their own by dramatizing it. They drew pictures to illustrate it so they would *know* what they were buying. They have estimated the answer and now have a guide by which they can judge whether or not the answer is reasonable.

Determine what is to be found

"What would you need to know if you were Mike?"

Write the question beneath the statement just as the volunteer dictates it.

Mike bought two loaves of bread at 27¢ a loaf and a gallon of milk at 45¢ a half-gallon. He bought two pounds of oranges for 36¢ a pound. How much will it all cost us?

This problem has now become *their* problem.

Find as many ways as possible to solve the problem

In choosing a volunteer to solve the problem, ask him what method he will use. If he says he is going to add to find the cost of the two loaves of bread (27¢+27¢), the two pounds of oranges (36¢+36¢), and the milk (45¢+45¢), encourage him, and then say to the other children, "While John is solving the problem by adding, can anyone think of another way to solve the problem?" Another volunteer may suggest multiplying, and he will use multiplication at the chalkboard in finding the cost of the loaves of bread, the two pounds of oranges, and the milk (27¢×2; 36¢×2; 45¢×2).

The children at their desks will watch the work being done at the chalkboard. If they see the need for a correction, they may step forward quietly and help their classmate.

Each volunteer, upon completion of the work being done at the chalkboard, will explain his work, calling upon the children who wish to comment.

Helpfulness and a courteous attitude are taught just as anything else is taught. Such remarks as, "Can we help Sharon? She did the work for all of us so let's help her if we can. Thank you for helping us, Sharon," are factors in establishing an atmosphere free from fear and blame and are conducive to exploration and the use of initiative.

Check the answer with the estimate

Thus, the children see that estimating the answer provides the security of knowing whether or not their answer is reasonable. Of paramount importance is the fact that estimation assures the pupils of seeing the problems as a whole. It is one of

the most valuable things we can do in arithmetic.

Continue exploration by asking the children again if they can think of additional ways to find the answer. *Be sure to give them time to think.*

"We added and we multiplied. Can you find another way?"

Never rule out a method even if you know it is wrong. Let the children try it. They just might have thought of something that may not have occurred to you! Join the children in their effort to try the method. If it fails, compliment them for having tried.

If you feel that this group is ready, and time permits, you may wish to ask, "What else might we want to know in this problem?" thus taking them into two- and three-step problems.

This group will then go to their desks to complete their daily assignment while you take on a second group of children, using another problem taken from their experience.

Pupil projects

After you work with groups of your pupils two or three times a week for approximately a month, they may be ready to continue on their own projects. Perhaps they are interested in shopping for a coming holiday. Other suggested projects around which they can write problems very real to them are:

Grade 3	*Grade 4*
pets	Brownies
parties	Cub Scouts
dolls	Camp Fire Girls
outdoor games	4-H Club
	pets
	sports
	bird clubs
	rock clubs

Grade 5	*Grade 6*
Boy Scouts	clubs
Girl Scouts	construction work (building)
Camp Fire Girls	sports
4-H Club	buying and selling
lumbering	gardening
construction (building)	hobbies
ball clubs	movie stars
secret clubs	
gardening	

The Stokes series, *Arithmetic In My World*,* gives basic work centered around units of particular interest to children. Social studies provides excellent sources for word problems.

Problems may be typed or written by the children on 5×8 cards in class or/and in individual scrapbooks. The scrapbooks can be made by using wrapping paper. A sewing machine can be used to sew along one edge to form a binding. Advertisements cut from newspapers, magazines, or shopping guides may be pasted into the scrapbooks or onto the cards. Sketches drawn by the pupils could also be used. The cards could be filed under given topics in a box. The problems on the cards or in the scrapbooks may be used for classwork, for individual work, or by committees working on these projects.

Grouping must be kept flexible after the first half-dozen class periods. By then the children will have lost much of the fear they may have had, will be relaxed, and will be eager to try different solutions. A unit on geometric shapes, cost of material, etc., may evolve from a sixth-grade project of actually building a small playhouse for younger children of the neighborhood. The natural basis for grouping will now become units of interest to the children and will be based on *real* participation.

* C. Newton Stokes, *Arithmetic In My World* (Boston: Allyn and Bacon, Inc., 1958), grades 1–9.

Familiarity with measurement

A study of beginning first-grade children's concepts

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Many educators feel that measurement is an important part of the elementary child's mathematical experiences because a great many in-school and out-of-school activities involve measurement ideas. Even though number ideas are independent from measurement ideas, the use of measures can give concrete meaning to many number abstractions. If measurement is to be included in the curriculum of the elementary school, it is necessary to know what facts the beginning first-grade child knows about measurement so that instruction can begin in appropriate places.

Purpose of the study

The purpose of this investigation was to ascertain beginning first-grade children's familiarity with measurement and to determine whether certain selected variables have any effect on this familiarity. The major tasks of the study were:

1. To discover to what extent children are familiar with measurement when they begin first grade.
2. To use the analysis of variance and the t test between the means to test subhypotheses of the following null hypotheses statistically:
   a. There is no significant difference in familiarity with measurement of beginning first-grade children within and among the chronologically younger, middle, and older groups.
   b. There is no significant difference in familiarity with measurement of beginning first-grade children within and among the lower, middle, and upper socioeconomic groups.
   c. There is no significant difference in familiarity with measurement of beginning first-grade children within and between the boy and the girl groups.
   d. There is no significant difference in familiarity with measurement of beginning first-grade children within and between the groups of children who have attended kindergarten and those who have not attended kindergarten.
   e. There is no significant difference in familiarity with measurement of beginning first-grade children within and among the groups of children of low, average, and high mental ability as measured by a readiness test.

Procedure

The scope of the study was limited to 150 beginning first-grade children who were attending the public schools of Muncie and Kokomo, Indiana. Three schools were used in each city. All of the children in the chosen schools were used in the study except those who were repeating the first grade or had repeated kindergarten, two children who were absent during the interview time and follow-up time, and one child with whom rapport could not be established.

The technique used for gathering the data for this investigation was the per-

sonal interview. Each child was interviewed by the author. The interview guide used was *The Test of the Pre-School Child's Familiarity with Measurement* devised by Josephine MacLatchy and Cecil Swales and adapted and expanded by the author. All of the interviews were completed by the end of the first month of school of the fall semester, 1960.

The data were placed on IBM cards so that the following statistical analyses could be made on the 650 electronic computer:

1. An analysis of variance with each variable group
2. A *t* test between the means of each possible combination of the subgroups of the selected variable groups
3. Percentages of correct responses to each item for the total group interviewed
4. Percentages of correct responses to each item for the subgroups within the selected variable groups

Findings

The following is a summary of the findings:

1. Examinees from 75 to 79 months-of-age possessed significantly more familiarity with measurement than did examinees from 71 to 75 months-of-age.
2. Examinees from 79 to 85 months-of-age possessed significantly more familiarity with measurement than did the examinees from 71 to 75 months-of-age.
3. Examinees from 75 to 79 months-of-age did not differ significantly from examinees who were 79 to 85 months-of-age in their familiarity with measurement.
4. Examinees from the upper socioeconomic strata did not differ significantly from examinees in the middle socioeconomic strata in their familiarity with measurement.
5. Examinees from the upper socioeconomic strata possessed significantly more familiarity with measurement than did examinees from the lower socioeconomic strata.
6. Examinees from the middle socioeconomic strata possessed significantly more familiarity with measurement than did examinees from the lower socioeconomic strata.
7. Male examinees did not differ significantly from female examinees in their familiarity with measurement.
8. Examinees who had attended kindergarten did not differ significantly from examinees who had not attended kindergarten in their familiarity with measurement.
9. Examinees who rated "superior" or "high normal" on the *Metropolitan Readiness Test* or "definitely ready" on the *Science Research Associates Readiness Test* possessed significantly more familiarity with measurement than did examinees who rated "average" on the *Metropolitan Readiness Test* or "probably ready" on the *Science Research Associates Readiness Test.*
10. Examinees who rated "superior" or "high normal" on the *Metropolitan Readiness Test* or "definitely ready" on the *Science Research Associates Readiness Test* possessed significantly more familiarity with measurement than did examinees who rated "low normal" or "poor risk" on the *Metropolitan Readiness Test* or "probably not ready" or "definitely not ready" on the *Science Research Associates Readiness Test.*
11. Examinees who rated "average" on the *Metropolitan Readiness Test* or "probably ready" on the *Science Research Associates Readiness Test* possessed significantly more familiarity with measurement than did examinees who rated "low normal" or "poor risk" on the *Metropolitan Readiness Test* or "probably not ready" or "definitely not ready" on the *Science Research Associates Readiness Test.*

Conclusions

On the basis of the findings of the present investigation, the following conclusions were drawn for the population tested and within the limits of this study:

1. Age seemed to contribute to the child's familiarity with measurement, the younger children being less familiar with the measurement items than were the older children.
2. Socioeconomic status seemed to contribute to the child's familiarity with measurement, the lower-socioeconomic-status children being less familiar with the measurement items than were the middle- or upper-socioeconomic-status children.
3. Mental ability as measured by a readiness test seemed to contribute to the child's familiarity with measurement, the children in the low-mental-ability group being less familiar with the measurement items than were the children of the higher-mental-ability groups.
4. Sex and attendance or nonattendance in kindergarten did not appear to contribute to the child's familiarity with measurement.
5. There seemed to be a wide range in the child's familiarity with measurement, as indicated by a range of 22 to 84 items answered correctly. The child's familiarity with measurement was greater when the terms were used in context than when the terms were treated as isolated factual situations.
6. Children seemed to be more familiar with money than with any other area of measurement.
7. The present investigation found children less familiar with measurement than previous studies indicated.

Implications

Relative to the conclusions of this study the following implications seem appropriate:

1. It can be implied from the data that the older the child is, the greater is his familiarity with measurement. Since familiarity with measurement can be considered a factor in the child's readiness to enter school, it would seem important to determine the time when the child would profit best from formal instruction in measurement.
2. It can be implied from the data that this investigation further verifies the principle that maturity plays a part in the child's ability to learn, even though a difference in maturity between boys and girls did not affect the children's familiarity with measurement.
3. It can be implied from the data that there are factors at work, such as poor environmental background, which make the lower socioeconomic children less familiar with measurement than the middle and upper socioeconomic children.
4. It can be implied from the data that the children who have high mental ability as measured by a readiness test can proceed more quickly in becoming familiar with measurement than can children of lower mental ability.
5. It can be implied from the data that children coming to the first grade have great differences in their familiarity with measurement. This fact in turn implies that it is necessary to take individual differences into account when planning measurement experiences for children.
6. It can be implied from the data, that, even though attendance in kindergarten is meant to aid a child in getting ready for first grade, attendance in kindergarten seems to have no effect on familiarity with measurement.
7. It can be implied from the data that children have heard many of the terms used in measurement and know the general meaning of the terms in context, but that the children are not familiar with the meaning of the terms used as abstract or exact measures.
8. It can be implied from the data that

children are most familiar with the measurement items which involve things that the children use most frequently, such as time and money. This implication would further imply that children tend to be most familiar with things they use and with things which have meaning for them.

9. It can be implied from the data that there were some factors at work which made the percentages of correct responses less for the present investigation than they were for the previous studies.

Recommendations

The following recommendations are based on information gained during the present investigation:

1. The measurement ideas taught in the first grade should be examined in light of the child's familiarity with measurement when he enters the first grade. Perhaps some of the ideas now considered appropriate for the first-grade level could be considered part of the child's knowledge when he enters school.
2. Consideration should be given by the first-grade teacher to the individual differences of each of her children when planning the curricular activities involving measurement ideas. Even though the child's familiarity with measurement can be determined in a general sense, there will inevitably be children who are at various levels of understanding in their familiarity with measurement.
3. Teachers need to study the composition of their groups of children in terms of the age level, socioeconomic level, and mental-ability level before planning curricular activities involving the children's familiarity with measurement. Each of these factors should be considered in light of the findings of this investigation.
4. Measurement should be taught in meaningful situations in which the child can actually use the measure in question, so that the idea being stressed is placed on a concrete rather than an abstract level.
5. Rich measurement environments should be provided by the first-grade teacher with many concrete materials at hand, so that the everyday in-school measurement problems can be solved by the children in an intellectually stimulating setting.
6. Further study should be done to aid in the isolation of other factors which may affect the child's familiarity with measurement. Such studies might try to find the answers to the following questions: (1) Is there an optimum time in the child's development when he would profit most from planned exposure to measurement ideas? (2) Are there significant differences in the child's familiarity with measurement between children from urban and rural areas?
7. Further study should be done in developing additional instruments to isolate other factors which may affect the beginning first-grade child's familiarity with measurement.
8. Further study should be done to help isolate factors which contribute to the fact that no significant difference was found between children who attended kindergarten and children who did not attend kindergarten. Some of these factors might be found in the following studies: (1) Are measurement ideas being considered as part of the kindergarten curriculum? (2) Are kindergarten children mature enough to have measurement ideas included in the kindergarten curriculum? (3) Are rich measurement environments being provided by the kindergarten curriculum?

Why do pupils avoid mathematics in high school?

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It must be pupils' unpleasant experiences with arithmetic in the grades that cause them to avoid mathematics in high school. Before failure of pupils was ruled out by "progressive" education, arithmetic caused more failures than any other subject in the elementary curriculum. Today a high percentage of pupils show no enthusiasm for arithmetic.

Need this be so? When well taught, arithmetic is one of the easiest of school subjects. The essential drill load is light. In addition there are 100 primary facts, half of them easy, such as $1+1$ or $1+4$, and 300 related decade facts, such as $11+1$ or $21+4$, to $39+9$. In subtraction there are 100 facts, no more. In multiplication to 9×9, we have 100 facts; and in division, 81 essential facts and then a scheme for the process of long division. Thus children must learn only 681 facts: $100+300+100+100+81$. In spelling there are 2,000 justifiable words for the grades, and spelling a word like "this" is more difficult than adding $1+2$. In reading, there are about 3,000 basic words, with indefinite extension through dictionary usage. The drill task in arithmetic, therefore, is simpler and easier than the drill work in either spelling or reading.

The values of drill

Drill in arithmetic beyond the fundamental processes is of doubtful value. This is shown by extensive research studies by Wilson, Wise, Woody, Charters, Dalrymple, Russell, and others.

Nevertheless most pupils in most schools today count instead of adding. The counting for answers in addition continues through the grades, through the high school, and into college. About fifty per cent of high school pupils fail to make a perfect score on a simple test in addition of whole numbers where no combination above $39+9$ appears. Most adults, if it is possible, avoid figuring of any kind.

This regrettable situation must be charged to poor teaching. The average teacher the country over doesn't know how many drill facts there are in the fundamental processes. He fails to organize the facts into a teaching program, although this should be an easy possibility.[1] He merely assigns pages in a text; he doesn't discover and follow the mental processes of the pupil and so he does not know what is going on in the pupil's mind. Left on his own, and in the absence of any real teaching, the pupil counts or guesses; he does not arrive at the mastery which brings confidence and satisfaction.

Problems and appreciations in arithmetic

Aside from the essential drill load there are two other phases of arithmetic, problem work and appreciation units.

Appreciation units are undertakings, preferably done by choice, on advanced or little-used processes, such as complicated fractions, decimals beyond money, ratio

[1] See *Teaching the New Arithmetic* by Wilson and others, Chapters 10, 11, 12, and 13 (2nd ed.; New York: McGraw-Hill Book Co., 1951).

and proportion, mensuration, business units involving percentage, units in algebra or elementary geometry, or interesting details in the development of the number system, such as presented in the article by Kreitz and Flournoy in the October, 1960, number of THE ARITHMETIC TEACHER. In the grades these can be managed as just-for-fun opportunities for pupils who like arithmetic and mathematics; they carry a forward look and are not part of the load in drill for mastery.

Meaningful problems of real value

Problem work as developed in the usual arithmetic textbook has little or no value. The real purpose of problem work should be the development of judgment in business; here figuring is merely a tool to aid in arriving at the best answer. In life there are real problems. For example, in buying a car, many questions arise and one may use figuring as an aid in getting the right answers. What make of car should one buy? What are the prices of various makes? What can, or should, the buyer afford? What is his income? How much service can the car provide? Is it needed as transportation to work or is other transportation convenient and economical? Should it be a new car or a used car? What is the cost of upkeep, of depreciation, of operation, of insurance, excise or other taxes, registration, and a driver's license? Judgment is needed at every turn.

Buying a car is a serious experience in life, and it could easily be used as a problem unit in school. It could profitably use the time of an arithmetic class for two days each week for several weeks. One capable teacher found the class sufficiently interested to pursue a unit on buying a car for a full half-year, two days each week. Many an adult knows that buying a car can occupy his attention for several months as he weighs and figures different possibilities. If he is buying a used car or trading in his car on another car, the difficulties of the problem are increased.

Buying a car as a problem for upper-grade pupils may be referred to very properly as a *functional problem unit.* Many such units have been developed, and the suggestion has been made that such units replace entirely the usual valueless list of isolated text problems.[2]

Open any sixth- or seventh-grade text in arithmetic and notice the trivial character of the problem work. It appears to be just something to figure on without real purpose. On a single page of a sixth-grade text, pupils are asked to figure in ten different situations: average football scores, money left after buying a shirt, amount of money lost out of a purse, weekly allowance total for a year, dividing pay for digging a ditch among fifteen boys (not a likely situation), cutting 24-inch lengths from a 30-foot canvas, putting up a 50-foot clothesline, length of rope left after cutting off a piece, knitting squares for a Red Cross blanket. No one of these ten text problems presents a real problem situation to a class; they are trivial opportunities for figuring. No one really cares what the answers are. They are really disguised drill: $144 \div 9$; $\$11.23-\$6.98$; 72¢ − 22¢; $\$35 \times 52$; sum of 69¢, 5¢, and 3×9¢; $\$6 \div 15$; 30 ft. ÷ 24 in. (or 2 ft.); $50-(20\frac{1}{2}+16+10\frac{2}{3})$; $18\frac{1}{4}-2\frac{7}{8}$; $168-(7 \times 23)$.

In life outside school people figure only when they want the answer, and they figure only when there is a background of experience.

Bringing judgment into problems

Some textbook writers have slightly improved the problem work by the use of "unified situations." This is a step in the right direction, but it doesn't go far enough.

It is, however, an easy matter to change a "unified situation" to a problem calling for judgment. In a text for the sixth grade under the heading "Going to Market," there is a unified situation with seven

[2] See an extended list of functional problem units in Chapter 26, *Teaching the New Arithmetic.* See also *Education* for Feb. '37, Apr. '41, Apr. '45, Feb. '49, and Apr. '51.

simple figuring opportunities or so-called problems: (1) cost of 8 oranges at 60¢ a dozen; (2) cost of 2 cantaloupes, each $4\frac{1}{4}$ lb. at 9¢ a pound; (3) cost of $4\frac{1}{2}$ lb. roast at 52¢ a pound; (4) cost of $\frac{1}{2}$ lb. of cherries at 39¢ a pound; (5) cost of 4 cans of tomato soup at $12\frac{1}{2}$¢ a can; (6) cost of $1\frac{1}{2}$ doz. eggs at 55¢ a dozen; (7) cost of $\frac{3}{4}$ lb. of cheese at 65¢ a pound.

As presented, this assignment consists of seven trivial tasks involving figuring that emphasizes fractions. No one of the tasks is likely to have any particular appeal for pupils. The assignment is just "some figuring" as far as children are concerned. The teacher may try to motivate the assignment by giving grades or posting the names of the successful. But there is no call for judgment and no carry-over to real life.

Now instead of this formal and relatively profitless procedure, suppose that the class undertakes to buy a week's supply of groceries for a family. They might agree upon a family of four—father, mother, and two childrren, with sex and ages specified. The class could agree that the father is a factory worker with a known take-home pay each week, and by relating this money to the rules for a family budget, pupils could decide on the percentage of wages that could be spent for food. Other conditions could be agreed upon. For instance, the family might have a garden to provide a supply of potatoes, beets, turnips, tomatoes, and so forth, for part of the year. There should, of course, be local adaptation to make the situation especially meaningful.

This procedure is shaping up a real problem that calls for judgment and not just figuring. One sixth grade, in Lynn, Massachusetts, developed a family budget unit as its problem work and spent two days each week for the entire year on this problem with its many interesting ramifications. A problem scale test at the end of the year indicated superior results even on a problem scale of doubtful value in itself.

Judgment in business and in practical affairs—this should be the real purpose of problem work in arithmetic. This was the original purpose and determined the type of problem work in colonial days and in our early national period. It was in the early part of the nineteenth century that Colburn gave us his "number by development" with a mental discipline justification. There has been no essential change in a century and a quarter. Tradition is very strong. The pupils of today deserve something better. They deserve functional problem units that will develop judgment for real life problems.

Functional problem units

Functional problem units will fall into one of two classes:

General units based upon wise use of the family income, really centering around the family budget. Such units as buying food for the family, the clothing budget, installing a bath, the family car (new or used, one car or two, and so forth), life insurance for the family are basic. These units are designed to give pupils a basis for understanding and co-operating in a wise budget plan for the family.

Vocational units designed to bring judgment to bear on vocational undertakings by members of a family. Units might involve such situations as: (a) Should a dairy farmer, milking 20 cows, also attempt to raise hogs? If so, how many brood sows? (b) Should a dairy farmer sell calves immediately, within a month, or should he hold them until they are one or two years old? (c) What returns can be expected from sheep on a farm? Is it more profitable to carry a flock of breeders or would it be better to buy and feed spring lambs? (d) What size farm can one man manage profitably? (e) What extra equipment and expense do the various types of farming require, dairy farming, hog raising, sheep farming, cash crops?

Obviously the problems suggested above are appropriate for pupils from farm homes only. In every case there should be

adaptation and choice by the pupils to insure a background for judgment and for gathering essential data. Usually a vocational functional problem unit will be pursued by one pupil or a small group, seldom by an entire class.

Vocational units for city children may be less easily determined because of the absence of opportunities for participation in actual management or production. However one job can be compared with another, one business can be compared with another, causes of losses in a business can be noted, capital needs can be noted and compared. Most functional problem opportunities in a city relate closely to the family income and expenditures. But city pupils have developed banking units, savings units, units on going to a summer camp or making a trip. Any of these affords good opportunities for business judgment, the proper aim of problem work.

Essential objectives of arithmetic instruction

The two essential objectives of arithmetic work in the grades are perfect mastery of the fundamentals that make up 90–95 per cent of adult figuring, and judgment-forming functional problem units. Beyond these objectives are reference and appreciation opportunities, particularly for the brighter pupils, who look forward to mathematics as a field of special enjoyment and worthwhile opportunity. No one of these objectives is furthered by the usual textbook problems.

Two views of arithmetic

The ancient Greeks recognized two types of arithmetic—theoretical mathematics with the aim of mental development for the leisure classes, and practical arithmetic for the countinghouse and the market place—the aim being usefulness in life.

These two viewpoints on arithmetic survive in America today. There are those who posit mathematical theory as the chief aim in teaching arithmetic, and the justification in one form or another is mental discipline. On the other hand, there are others who base their arguments on social utility and urge that usefulness in life should be the chief objective in teaching arithmetic.

When these two viewpoints are presented to a group of school patrons, the vote is unanimous in favor of usefulness in life as the chief aim. On the other hand, the vote of a group of educators is likely to show a slight majority in favor of "intellectual development" (mental discipline) as the chief aim of arithmetic. Are most educators lacking in practical experience? Both groups agree in theory that any good teaching should be based upon understanding and motivation.

The attitude of pupils toward mathematics in high school can be changed and it has been changed in school systems where good teaching accomplishes the legitimate aims of arithmetic in the grades, namely (1) perfect scores in the essential drill processes following good understanding and adequate motivation, (2) the development of business judgment opportunities through functional problem units, and (3) appreciation units, adjusted to ability and choice.

In the subject of arithmetic, teachers have a splendid opportunity for service through good teaching. But good teaching requires more of a teacher than page assignments, testing results, and recording grades. Teachers need to get into the real purposes of arithmetic, and with a proper basis in meaning and motivation, help pupils to realize the true purposes of the subject.

Guidance counselors are encouraging more able students to take more than the required number of courses for graduation—18 to 20 or more units—instead of the traditional 16.—*From "Schools in Our Democracy," Office of Education, U.S. Department of Health, Education, and Welfare.*

Projects make mathematics more interesting

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In a recent survey the U.S. Office of Education asked mathematics teachers to rate according to preference a number of items of equipment which they had used for mathematics classes. Mathematical models led the list.

This indicates the high regard most practicing teachers of mathematics have for materials that illustrate in a concrete way the more demonstrable principles of our favorite subject. Many of the commercial models which do this, however, are quite expensive and beyond the reach of the budget of smaller schools. On the other hand, classroom teachers have for years been turning out numbers of creditable models as by-products of classwork in mathematics.

For a number of years we have been using a mathematics project summary for one of the terminating activities in grade 8. The results have been so encouraging and the pupil outcomes, both in terms of knowledge, attitudes, and basic concepts as well as in terms of the models, have been so good that we would like to call attention to them. Possibly here is one answer to the problem of the skyrocketing costs of modern education. We only wish to point out some of the affirmative phases of this highly interesting activity. We realize of course that there are many teachers who regularly do this sort of thing within the curriculum, yet we have found many more teachers who do not use such activities.

Naturally, there are many objections raised by the skeptical teacher. Among these are:

"Takes too much time. Interferes with time allotments."
"Models are junky and too hard to store."
"Prepared commercial demonstrations are more practical."
"Models are lacking in pupil appeal."
"The mathematical approach is vague and confusing to pupils."
"Pupils do too much copying and not enough creative work."
"Projects clutter up the math room. Pupils play with them."
"Activities are expensive to the pupil."
"Projects are false or poorly constructed."

Teachers who are using the project activity during the year take a different view. Most of them feel that it does not take a significant amount of time from the curriculum because most of the assignment is done as homework. These teachers feel this is a highly productive kind of homework because it offers a chance for creative work as well as a good situation for recitation.

How do the pupils feel about such an activity? No doubt there are many who do not see any value in it at all. By careful observation and casual interview we obtained comments such as the following:

"My project took about two hours' work last night."
"I would have been doing something else anyway."
"My father said it was a waste of time."
"I enjoyed the shows. You learn better from them."
"My mark was C – anyway."

"Boys are better at making machines. The girls study and plan more [a boy's comment]."

"We still do the daily work anyway."

Regardless of adverse comments, we make a careful appraisal each year of the outcomes of the "extra-project" activities as well as of the products themselves. This keeps the whole activity from straying away from the base of pupil interest, which is the motivating force.

Unless the project is an out-and-out toy with definite recreational possibilities, we have found the highest pupil interest appeal in projects that show the clearest mathematical implications. For example, Janet chose to construct a demonstration chart showing the relation of the laws of probability to genetics. She used Mendel's series of experiments with sweet peas as a basis. Although one might not suspect it, the pupil appeal of this project was very high. It was almost as if young people were amazed to think that inheritance of definite characteristics could possibly follow a definite (but not absolute) pattern.

The binary system of numbers, which is each year introduced to the grade by a number stunt, formed the basis for three projects.

David applied the binary system to a series or closed set of chemical elements and isolated the chosen element in six sortings by using a primitive system of notched and punched cards. Thomas prepared an exhaustive set of punched cards for a set of 255 numbers. He wrote the binary number on each card and related it to the shorthand base 8 during his demonstration to the class. Guy built an actual single-cell sorting machine which was operated mechanically. During the course of his demonstration to the class he described the characteristics of the binary system as applied to typical electric circuits. (None of this had been taken up in class.) If he was at all nervous during his demonstration, one never knew it. The chosen sample alone remained on the pin as it was designed to do. This project was voted the best of the year by the pupils and Guy, a "B" pupil at his best, was very happy about that outcome.

A project by Philip was outstanding both for pupil appeal and toy value. With the help of his father, Philip constructed a probability game board which consisted of a feeder trough for $\frac{3}{4}$-inch marbles that led into a field of steel brads preceding a series of 20 terminal troughs. The whole project was built from plywood and pupils were constantly trying it out before and after school.

Another project, built by Stephan, consisted of a series of concentric circles sawed out of masonite, each with a hole drilled in the center and arranged from larger to smaller on one of three nails sticking up out of a board. (Stephan got the idea from a small game someone had given him for Christmas.) The object was to remove all the discs, one at a time, to another nail without placing a larger disc over a smaller one at any time. While other pupils were playing with his probability board, Stephan was trying to beat the time on this combination shift. His best time for seven discs was four minutes and fifty seconds, and this stands as the present record in our school.

All the projects were not toys. Some were demonstrations of the correlation between mathematics and science. Chester constructed a device by using a cylindrical fruit juice can that contained exactly 45 cubic inches. This he placed in the center of a larger cut-off can with a depth of about three inches. The outer vessel had a cubic content of fourteen inches around the center one. Chester selected a rock and shaped it (irregularly) so that when submerged it displaced exactly enough water to fill the outside can to the brim. The fact that the rock displaced fourteen cubic inches of water gives its true volume according to the principle discovered by the famous Greek mathematician Archimedes, "A submerged object displaces an equal volume of water."

Paul constructed two four-inch cubic dice from hardwood. Using these, he demonstrated the odds on various numbers coming up on any given roll.

The Pythagorean theorem was a popular choice and was demonstrated several ways. Paul constructed a basic triangle three by four by five inches with raised squares on each side. Using glass marbles one inch in diameter, it was easy to show that the nine marbles on the base square added to the sixteen marbles on the altitude square just exactly filled the square on the hypoteneuse with 25 marbles. The bright-colored marbles added greatly to the pupil appeal of this project.

Carole demonstrated the Pythagorean theorem by constructing a toy house and a series of small ladders ranging from three to fifteen inches to show the multiple series of three-, four-, and five-inch triangles.

Several projects were of the research type. Two of these deserve honorable mention. David wrote, edited, illustrated, and typed a 30-page copy of a "Book of Interesting Mathematics." It was just that. During the course of his research he came upon such interesting and famous algorithms as Euclid's L.C.M. and Eratosthenes' Sieve for Primes, as well as Pascal's triangle. Angela wrote a scholarly presentation of three methods of finding the volume of a sphere (formula, limits, and graphic). Her blackboard demonstration of these three methods to an accelerated seventh-grade class clearly demonstrated her fitness for her chosen profession, teaching. This was an outcome we had not expected, and it could well be the most important one.

There were many other excellent projects. We have only mentioned ones that seem to show a trend. Naturally there were a number of oversimplified, poorly constructed, and even basically illogical demonstrations. This is always to be expected. Some projects were copied verbatim from textbooks. Others were poor copies of projects submitted earlier and were accompanied by highly irrelevant demonstrations. Many of these were from pupils of somewhat less than average ability either in mathematics or construction. In many cases, pupils asked for more time to make improvements in their work and returned with a fairly well-done piece of work. Every project was kept, and all pupils were assigned order-numbers in which to demonstrate their projects to the class. Classmates were quick to point up the falsities in any questionable presentation.

All projects were looked at, examined, discussed, and manipulated, but they were *not graded* as such. We have found this fact to be of considerable importance in the success of this activity. Another positive contributing factor was that *the project was not required.*

The preproject planning stage of this whole activity was important. During the course of the year's work, the teacher used several demonstrations made in preceding years. Each time these were used the teacher was careful to give the pupil who originally designed the project full credit. This is possible because a record is carefully kept by means of a series of descriptive booklets filed from year to year. Each pupil writes a description of his own project in a small standard-sized booklet for easy filing chronologically. It thus takes but a minute for the classroom teacher to connect the right pupil to the project for future discussions. A pupil derives great pleasure from knowing that something he made is being used daily or at least periodically during the course of classroom work. Not only that, it becomes a simple matter of replacement, usually with needed improvements, when a demonstration model begins to show signs of wear and tear. Pupils of high constructive but low creative abilities enjoy making these improvements.

For a number of years, projects were required from every pupil. It became gradually apparent that the bulk of the work was of the one-night, minimum-assignment variety. A much better selection was

obtained when we adopted the following organization.

1. Projects were required from only the "A" and "B" pupils. Immediately the picture changed. Pupils who were just passing requested the chance to make a project and lift their grades. The "A's" and "B's" felt honored at their selection and put forth real efforts to produce superior projects.
2. The general call for projects went out several months in advance of the deadline, usually May 1. Good pupils were quick to grasp this advantage in planning time. Research materials were gathered and organized well in advance of the final construction date.
3. Seventh-grade classes with excellent potential mathematics ability were invited to sit in on the eighth-grade demonstrations in May and June. We would commonly double a seventh- and eighth-grade division to watch the demonstrations. This gave the incoming eighth-graders a preview of the whole project activity. It usually motivated superior pupils' planning.

In conclusion, to answer some of the criticisms which have been raised, we find pupils as well as teachers have clearly expressed themselves.

Pupils say

"We would rather watch somebody we know do his project than have the teacher do a bought one."

"The bought ones are better, but they cost a lot."

"Boys and girls like to make useful things."

"It's nice to know your project will always be used."

"Pupils' projects do not cost the school very much, and they can always be made better next time."

"I like projects—they make mathematics more interesting."

"I think I learned more from the projects than from the examples."

"Colors make things clearer and more reasonable."

"My project wasn't so good, but I thought some of the others were very good."

"Some pupils who did not do very well in math did some very good projects."

Teachers say

"Whatever the pupil project lacks in design, construction, and durability, it more than makes up in pupil appeal and enthusiasm."

"Pupils like to try each other's projects and freely criticize them as to their validity."

"A few pupil-made demonstration models are actually superior to commercial items."

"Projects brighten up the classroom during the last two months of the school year. They keep up interest in the subject till the very last."

"There is no doubt that pupils learn by doing."

Finally, it must be said that many fine pupils arrive in Grade 8 with a firm dislike for the subject of mathematics. Regardless of how or why it developed, the dislike is evident to any experienced teacher. The project activity is just one more interesting way of putting the subject across to this type of pupil. Many pupils and a few teachers frankly admit that an activity of this type is a richly rewarding experience, especially if it goes off at all well. We cannot stress too much the importance of the teacher's own enthusiasm for the activity. This becomes very evident to the pupils during the year as previous projects are being used in the classroom work.

Even if the year's crop of projects is not a huge success, what has been lost? An hour or two of class time for demonstration at the most. At the very least, you will get one or two models you can use. Even the poorest projects often have unexpectedly good outcomes. There is always the hopeful thought that even if you have reached only one pupil who was not responding to the academic approach, it was all worthwhile. See if you don't agree.

Dividing by zero

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The questions surrounding the idea of dividing by zero are recurrent and perplexing. Attempts to answer the questions are often based on semantic confusion between "nothing" and "zero," but zero is not nothing. As we know, zero is a number, in particular it is the cardinal number of the empty set. Zero is the number that tells us *how many* we have of something when we don't have any at all. Nothing is the state of nonexistence itself. Most dictionaries lend aid and comfort to this confusion by equating the two concepts in definitions. An error is often made by assuming that the opposite of nothing is anything. It would seem much more reasonable to let everything or something be the opposite of nothing.

Other attempts to answer questions about division by zero can be refuted on purely mathematical grounds.

Some invalid arguments

Let us look at several answers, all of which will be shown to be unacceptable, and then discuss the right answer.

Attempt 1: "Dividing a number by 0 is dividing by nothing, and if you don't divide by anything, you don't get anything, so the answer is 0."

$$\text{Example: } \frac{6}{0}=0.$$

Attempt 2: This is the same as the first attempt, except that the conclusion is, "If you don't divide by anything, the number stays the same, so the answer is the number you started with."

$$\text{Example: } \frac{6}{0}=6.$$

Attempt 3: "When you divide a number by 0, you get 1."

$$\text{Example: } \frac{6}{0}=1.$$

Attempt 4: "A number divided by zero is infinity, because if $\frac{6}{0}$ is to be a number, it should be a unique number."

Refutations

A mathematical refutation of the first three attempts depends upon the fact that we define a fractional number $\frac{a}{b}$ to be equal to a number c if, and only if, $a=bc$.

For example, $\frac{6}{2}=3$, since $6=2\times 3$. Applying this definition to the second attempt, suppose that $\frac{6}{0}=6$. Then

$$\frac{1}{6}\times\frac{6}{0}=\frac{1}{6}\times 6.$$

Thus, $\frac{1\times 6}{6\times 0}=1$. Thus, $\frac{6}{6\times 0}=1$. Thus, $\frac{6}{0}=1$.

But we assumed that $\frac{6}{0}=6$. This is inconsistent with our assumption and so $\frac{6}{0}$ cannot be 6.

On the other hand, suppose $\frac{6}{0}=1$. Then, $\frac{1}{6}\times\frac{6}{0}=\frac{1}{6}\times 1$. Thus, $\frac{1\times 6}{6\times 0}=\frac{1}{6}$. Thus, $\frac{6}{6\times 0}=\frac{1}{6}$. Finally, $\frac{6}{0}=\frac{1}{6}$ · Again, we have a contradiction since we assumed $\frac{6}{0}=1$. Therefore, $\frac{6}{0}$ cannot be 1.

In order, the definition yields $6=0\times 0$, $6=0\times 6$, and $6=0\times 1$. These conclusions are clearly unacceptable.

The fourth attempt is not completely adequate, but it does contain the real crux of the matter.

The crux of the matter

It can be argued that if $\frac{6}{0}$ is going to have a value, it should be infinity, usually written by mathematicians as ∞. This can be seen by examining $\frac{6}{x}$ for values of x getting closer and closer to 0. For example,

$$\frac{6}{1}=6,\quad \frac{6}{\frac{1}{2}}=12,\quad \frac{6}{\frac{1}{10}}=60,\quad \frac{6}{\frac{1}{1000}}=6000.$$

It is seen that as x gets closer and closer to 0, $\frac{6}{x}$ gets larger and larger without bound, i.e., $\frac{6}{x}$ gets as large as we like. Mathematicians express this by saying that as x approaches 0, $\frac{6}{x}$ "becomes as large as we want," "increases without bound," or "approaches infinity." They also say that "the *limit* of $\frac{6}{x}$ as x approaches 0 is infinity." This is abbreviated as

$$\lim_{x\to 0}\frac{6}{x}=\infty.$$

It is this process of examining limits which leads to the subject of the calculus in higher mathematics.

Another way of seeing this is to go back to the definition of division as "repeated subtraction." Thus, when we say $8\div 2=4$, what we mean is that 2 can be subtracted from 8 exactly 4 times. Similarly, $8\div 1=8$ means that 1 can be subtracted from 8 exactly 8 times. Now what would $6\div 0$ give us by the definition? Since 0 can be subtracted from 6 as often as we like, i.e., an infinite number of times, we are led to the result, $6\div 0$ is infinite if our definition is to hold in this case as in all others.

Then the question arises as to whether it is correct to say that $\frac{6}{0}=\infty$.

The answer is yes and no; yes in a certain sense in higher mathematics, but no in elementary mathematics. The reason for the negative answer follows. In the fifth grade, when children do subtraction, they must be careful to avoid problems like $2-4$. This is because they are not yet acquainted with negative numbers. In the sixth grade, when pupils learn about negative integers, they can write $2-4=-2$. But in the fifth grade, $2-4$ is impossible because it would violate the principle of *closure.*

The principle of closure says that an operation (such as subtraction) can be performed on two numbers only when the result is a number in the set of numbers in which one is working. (In the fifth grade, this is the set of whole numbers). This means that our set of numbers is kept *closed*, or restricted, under the operation

in question. So it is when we come to division. We are *not allowed to say* $\frac{6}{0} = \infty$ unless ∞ is a number in the set of numbers with which we are working. But the sets of numbers used in elementary school and even all the way through high school to the college level do not include ∞ as a number. We can count 1, 2, 3, 4, $\cdots$ as far as we like, and all these numbers are natural numbers, but we never reach ∞. Therefore, ∞ is *not* a counting number nor is it a fractional number nor even an irrational number.

In some college-level courses, infinities of several types are adjoined to the number system, but this is done systematically through the study of infinite sets. The resulting new arithmetic is structurally quite different from our familiar arithmetic.

Since relatively few students will specialize in mathematics to the extent of reaching the arithmetic of the infinite, it is best not to get any more involved in these fairly esoteric questions than is absolutely necessary at this time. The hurdle to be overcome in treating $\frac{6}{0}$ is seen to be much greater than that in dealing with $2-4$ at the elementary level before negative numbers are introduced.

What can we say?

The right answer to the question is contained in the foregoing paragraph. Just as in grade five, before negative numbers are introduced, we must say that $2-4$ is an impossible or impermissible problem, so *we must say throughout elementary- and secondary-school years that* $\frac{6}{0}$ *is an impossible or impermissible problem.*

It should be noted that all of the preceding analysis holds equally well for $\frac{5}{0}$, or $\frac{2}{0}$, or $\frac{-3}{0}$, or any other case of division by zero. Here $\frac{6}{0}$ was used only as a concrete example.

The case of $\frac{0}{0}$ is somewhat special. For this expression, it would seem reasonable to assume that $\frac{0}{0} = 1$, because anything divided by itself is one. But here again, we get into mathematical difficulty. Assume that $\frac{0}{0} = 1$. Then

$$\frac{0}{0} \times 2 = 1 \times 2. \quad \text{Then} \quad \frac{0 \times 2}{0} = 1 \times 2.$$

Then $\frac{0}{0} = 2$. But we have said that $\frac{0}{0} = 1$. This is an unsatisfactory state of affairs because we want $\frac{0}{0}$ to have a unique value, and so $\frac{0}{0}$ cannot be unity.

Let us notice that $\frac{0}{0} = 0$ does not lead to algebraic inconsistency within the system of numbers. It is impossible to refute this single case in the same way that we refute attempted definitions of $\frac{n}{0}$, when n is not itself equal to 0. However, it is found in higher mathematics that a definition of $\frac{0}{0} = 0$ would be unsatisfactory because it would not agree with results of the limit process and would lead to discontinuities in several elementary types of functions. What this means is that some functions which should have smooth graphs will

have "holes" in them if we agree to let $\frac{0}{0}=0$. For a particularly simple example, try to graph $\frac{x}{x}$ for all real values of x. If $\frac{0}{0}=0$, there will be a hole in the graph for the value $x=0$. The graph would seem to suggest defining $\frac{0}{0}=1$, but we have already seen that this is not satisfactory.

Therefore, we again emphasize that division by 0 is undefined—even when the dividend is 0.

The metric system IS simple!

RICHARD H. PRAY *El Centro, California*

During 1959–60 Mr. Pray taught in the elementary and high school in St. Johns, Arizona. Presently he is enrolled at the University of California, continuing his major in mathematics and working to complete his Master of Science degree.

With all the clamor about the United States being "first" in space, science, and other related areas, the people of the country have already relegated themselves to one technological "last." Every nation in the world with the exception of the United States and England has switched to the easier metric system of measurement.

Radical change of any sort is difficult, particularly in a democracy where the people must agree to it. However, through sane presentation, the children now in school could be shown that it *is* a better system. Herein lies the difficulty.

Schools and textbooks spend too much time stressing the relationships between the metric and English systems. Students are told that the metric system is simple and therefore easier, but then are assigned tedious exercises converting from one system to the other and back again. This involves remembering "one meter equals thirty-nine and thirty-seven hundredths inches," as well as multiplying and dividing decimals (which is certainly not the favorite pastime of most pupils).

The point that the writer wishes to make is that *the metric system should be taught as a system of measurement—not as a comparison* to our own system.

Beyond the incidental fact that a meter is slightly more than a yard, *conversion from the metric to the English should be eliminated entirely.* The student should be provided with a metric ruler and given ample opportunity to take measurements, manipulate them, and come up with answers in (and left in) metric form. In short, he should use the metric system for measuring if he is to learn it and regard it favorably.

The object is not to enable the student to be able to convert, but to allow him to compare the two systems as separate entities—without confusing the issue by switching back and forth between them—and to let him decide which is the easier to handle.

More on divisibility by seven and thirteen

GEORGE S. CUNNINGHAM *Concord, New Hampshire*

Mr. Cunningham is director of mathematics education in the New Hampshire State Depatment of Education. He is also a part-time lecturer in education and mathematics for the University of Maine, Extension Division at Portland, Maine.

In his article, *Divisibility by Seven and Thirteen,* in the November, 1958 issue of THE ARITHMETIC TEACHER, Mueller derives and states rules for testing divisibility of whole numbers by seven and by thirteen. These rules are presumably designed to take less time than division, since they are in answer to Yearout's earlier statement that there are no simple rules for divisibility by seven.

Suppose we test 1,123,456,789 by Mueller's rules and then by simple division. The rules follow:

Test for divisibility by seven

A whole number is exactly divisible by seven if the difference between twice its unit-digit and the number formed by its non-unit digits is exactly divisible by seven.

Test for divisibility by thirteen

A whole number is exactly divisible by 13 if the sum of four times the units-digit added to the number formed by its non-unit digits is exactly divisible by 13.

The procedure of Mueller's rule for divisibility by seven and by thirteen would run like this:

By seven: 112,345,678
−18

112,345,6$\cancel{6}\cancel{0}$
−1 2

112,344,$\cancel{4}$
−8

112,33$\cancel{6}$
−12

112,2$\cancel{1}$
−2

11$\cancel{2}$,$\cancel{0}$
−4

7

By thirteen: 112,345,678
+36

112,345,71$\cancel{4}$
+16

112,345,8$\cancel{7}$
+2 8

112,348,$\cancel{6}$
+24

112,37$\cancel{2}$
+8

112,4$\cancel{5}$
+2 0

114 $\cancel{4}$
+16

13$\cancel{0}$ (Original number is exactly divisible by thirteen.)

In this operation of the rules for testing, it would seem that little, if any, advantage

is gained over testing by simple division. There is, however, a supplemental rule for rapidly reducing large numbers to three-digit numbers that are equivalent to the original number in their divisibility by both seven and thirteen, and which uses Mueller's procedures for testing the resulting three-digit equivalents.

Reduction rule

A whole number is exactly divisible by seven if the reduction number is exactly divisible by seven; and is divisible by thirteen ÷ if the reduction number is exactly divisible by thirteen. The reduction number is determined by the difference between the sum of the number formed by three digits on the right with the numbers formed by alternate three digits toward the left, and the sum of numbers formed by the second set of three digits with the numbers formed by alternate three digits toward the left. (By alternate three digits, it is meant to skip three digits, then include three, etc.)

The application of the reduction rule to our illustration 1,123,456,789 yields (789 plus 123) − (456 plus 1) = 912 − 456 = 455. 455 is easily determined by simple division or Mueller's rules to be divisible both by seven and by thirteen. We know from this divisibility that 1,123,456,789 is also divisible by both 7 and 13.

Proof of reduction rule

It is only necessary to note that 1,000,000 is equal to $7\times 13\times 10{,}989+1$ to see that N millions will have exactly the same divisibility by either 7 or 13 as N alone. We also note that 1,000 is equal to $7\times 13\times 11-1$. We see that N thousand will have exactly the divisibility of negative N. From these observations and the distributive law, we conclude that A billion, B million, C thousand, D will have exactly the divisibility (as far as 7 and 13 are concerned) as $(D+B)-(C+A)$. Since negative numbers have the same divisibility as their positive prototypes, we take the absolute difference on the test.

A method for checking addition

1. Add x numbers arranged in a column to obtain their sum c.
2. Add all but the last two numbers to get sum a.
3. Add the last two numbers to get sum b.
4. Continue to add the digits of a, b, and c individually until you obtain single digit numbers (e.g., for 12, $1+2=3$).
5. Record sum a.
6. Record difference of $(b-c)$.
7. Sum of steps 5 and 6 must equal 9 or 0, if the addition is correct.

An example:

8	Sum c equals 26.	$2+6=8$
6	Sum a equals 14.	$1+4=5$
9	Sum b equals 12.	$1+2=3$
3		
—		
26		

$$a+(b-c)=W$$
$$5+(3-8)=W$$
$$5+(-5)=W$$
$$0=W$$

Correct because $W=0$

Ronald Zimmerman
Doane College
Crete, Nebraska

The versatile number runner*

MARY MICHALOV *Chicago, Illinois*

Miss Michalov is a first-grade teacher at Rugen School in Glenview, Illinois.

The number runner is a simple arithmetic device that can be used with amazing success in the primary and intermediate grades to develop meaningful arithmetic concepts. The unbroken line of uniformly sized numbers and symbols lends itself to pupil discovery of many number relationships and patterns. The opportunity of handling the runner, of visualizing the arrangement of numbers in sequence, and of discovering number secrets results in keen understanding and in a photographic retention of number values, number facts and patterns, and arithmetic processes.

It is imperative that the runner be attached low enough on the wall in the primary grades that the children can handle it at eye level. It can be extended along one or more walls in the classroom to go as far as possible beyond 100. (See Figure 1.)

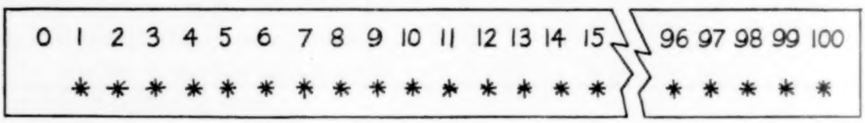

Figure 1

The runner can be remade annually by the teacher since much handling in the course of a school term soils it noticeably. The only materials required for its construction are a print set, or marking pen, and some 4″×30″ oak tag sentence strips. A star, a circle, or any form or picture stamp may be placed in one-to-one correspondence with each numeral. A form or picture stamp is preferable to mere lines, as used on a ruler.

* A condensation of this article appeared in the May, 1960 issue of *Illinois Education.*

Using the runner

The first-grade pupil will get a broad understanding of number relationships through the use of this device.

By touching the number symbols used, the child learns to count with understanding, visualizing that counting means taking one more object as he progresses along the runner. The counting may be developed in several ways: 1, 2, 3, etc.; 1 and 1 more are 2, and 1 more are 3, etc.; after 1 comes 2, after 2 comes 3, etc.

Before the numerals are learned, simple grouping can be developed all along the runner by several pupils simultaneously. "Enclose 2 stars with your hands (**), 3 stars, 5 stars, 4 stars, 1 star." The teacher can test several pupils in recognizing and showing number groups readily; while the actual enclosure of a given number of symbols with both hands gives the pupil a mastery of the problem assigned—kinetically, visually, conceptually, and retentively.

With no allusion made by the teacher, some pupil will discover that each symbol has a figure name above; and since most

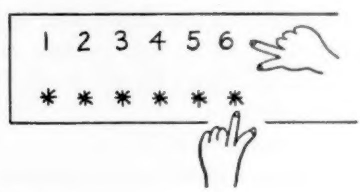

Figure 2

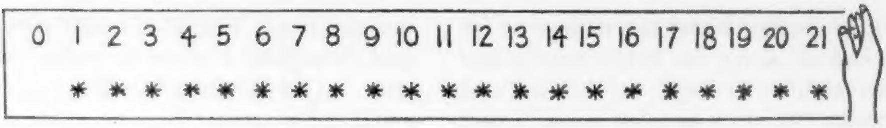

Figure 3

children have some knowledge of figures, they quickly recognize that there is a relationship between the symbol and the figure above (Fig. 2). When this discovery is made by a child, the teacher can draw the attention of the class to the relationships all along the runner and spend some time in studying the digit symbols and their names. "The first star's name is 1 (pointing to the star and its figure name above), the second star's name is 2, etc." Later on the teacher may ask pupils to find the stars whose names are 9, 15, 24, 53, etc.

When a large number of stars are counted, only one hand need be used to show that all the symbols from the first to the last one make the complete group. (See Figure 3.) There are 21 stars, or a group of 21.

Making discoveries

Discovery of many number arrangements can now be found and shown on the runner. "Look at the number runner and find some other interesting things." One child may go to the runner and find and count all the 40's, from 40 to 49; and quickly others will find the other decade groups. Another child will see the 0 numbers—10, 20, 30, etc.; and other children will find the twins—11, 22, 33, etc.;—and the repetition of the digits 0 to 9 in the different decade groups along the entire runner. They become fascinated with the game of finding new secrets on the runner device.

When they can count from 1 to 100 by touching the stars and recognizing their number names above, they are ready to count across the runner by 2's, 3's, and 4's. Insist that they use the left hand to show the distance traveled and the right hand to pick up the next group in progression. They may begin at any number and proceed successfully inasmuch as groups of 2, 3, and 4 are readily recognized and their number names are visible above. For example, counting by 3's can take 0, 1, or 2 as a starting point; counting by 4's may begin at 0, 1, 2, or 3.

Tricks for picking up 5's and 6's without counting can be shown by the teacher through blackboard exercises and practice in recognizing the number of boxes on the pages in their reading books. By covering the middle box in a group of 5 objects or pictures, the children will notice that there are 2 on each side of the hand, making a total of 4; and when the hidden one is exposed, there are 5 in all (Fig. 4). So to show 5 symbols on the runner, 2 are picked up, 1 hidden with the hand, and 2 more are shown to make a 5-group without counting. (See Figure 5.) Children develop this skill readily. To count by 6's, they merely take two groups of 3 stars and give the new sum.

At this stage of development subtraction in series can be introduced. One child

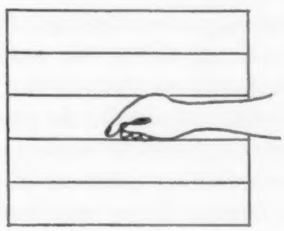

Figure 4

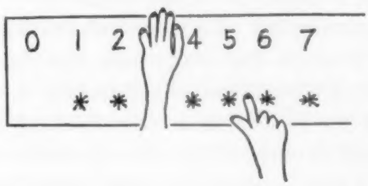

Figure 5

adds any number along the runner as far as he can or along the entire track; and another child may begin at the end and take away the same number back to the starting point. The terms and the concept of the two major arithmetic processes, addition and subtraction, thus introduced are visualized, pointed out, stated orally, and understood by the children. A good game is to tell a pupil to close his eyes, walk along the wall until he is told to stop, touch the runner, open his eyes, and begin to count by a given number from the number that he is touching, either adding or taking away as directed.

Counting by 10's must be supervised by the teacher, and several lessons may be necessary to fix this secret of the number runner. A child is asked to enclose a certain number of stars, such as 4, and to touch the fourth one. Another child counts 10 more stars aloud, touching each symbol as he gives its name. All note the number he points to at the end of the count (14). A third child repeats the process, adding 10 to the count, and all observe his stop figure (24). Continuing this exercise across the runner, the discovery is made that all the children at the runner are touching a number containing a 4. Then one child may review the problem from the starting point 4, independently. At subsequent lessons counting by 10's may be developed, using all the digits as starting numbers.

An occasional lesson for testing purposes is valuable to the teacher. A child may go to the runner and do anything he wishes. The teacher may request that each pupil do something different during this particular lesson. Later, exercises may be strictly oral, with pupils going to the runner only to prove those facts that are given wrong or those not known readily.

Counting by 11's is an interesting discovery since the stops are the twins—11, 22, 33, etc. One of my pupils chose to count by 11's from 1 as a starting point and did it correctly across the entire runner. When I asked how she learned to do that, she replied that her mother taught her the trick: that she should pick up 10 and then take 1 more to make the 11th one. $(1+11=12+11=23+11=34+11=45$, etc.)

To develop mastery of number sequence by name, exercises may be given in glancing at, or showing on the runner, answers to such questions as: What are the names of the 3 numbers that come after 5? After 9? After 32? Of the 2 numbers that come before 7? 11? This exercise serves as a basis for the addition and subtraction processes involved in oral and mental problem-solving.

Learning about addition and subtraction

Addition and subtraction can now be developed, involving the smaller digits (1–6) and the 10's. A child shows and gives the answers for such dictated problems as: $6+2+10+1+3+1$, or $98-10-1-3-10$. Boys and girls like to run up and down the number track. (A toy car can be held by the pupil making the trip.)

The next step in this dramatized number development is a combination of addition and subtraction exercises. A child can show and state answers for such dictated problems as: $6+2+10-3-10+5+5-2-10$.

Many new discoveries in the addition and subtraction facts can now be developed on the runner: $2+2=4$, $12+2=14$, $22+2=24$, etc.; $3-1=2$, $13-1=12$, $33-1=32$, etc. If a child knows one digit fact, he really knows the same fact for each decade group.

Solving problems

On "story-problem day," one pupil makes up a problem and another repeats it, gives the answer, and completes the number story by pointing out the number of symbols on the runner. Example of the second child's response to a made-up problem: "If you had 5 suckers in this hand and 2 in this hand, you would have 7 suckers, because 5 and 2 are 7"; and he shows 5 symbols with one hand and 2

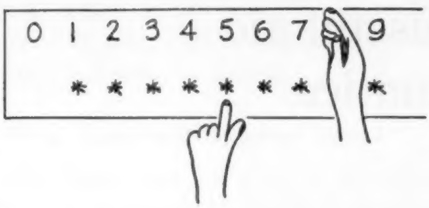

Figure 6

more with the other on the runner, to make 7 in all (Fig. 6). He understands the problem, he says it, he proves it, he visualizes it, he knows it, and he retains it.

One day each week (Friday in my room) can be used profitably for oral story problems. It provides the teacher an excellent opportunity to learn which of her pupils are adept in making and solving problems. Children frequently make advanced 2-step problems that the teacher must help them to answer in two separate statements. Example: (Problem) "If I had 10¢ in this hand and 5¢ in this hand, and I spent 2¢, how many would I have left?" (Answer) "If you had 10¢ in this hand and 5¢ in this hand, you would have 15¢ because 10 and 5 are 15; and if you spent 2¢, you would have 13¢ left because 15 take away 2 is 13." The pupil shows the two steps on the runner as he repeats the problem and solves it orally. This lesson may be varied by having the child who states the problem call on another child to answer it, or by having the teacher call on someone to do so. The latter method demands greater concentration by every pupil since no one knows who will be called upon for the solution.

During the spring months of the term, it is possible to write long problems on the blackboard, involving addition and subtraction using the small digits and the 10's, and the children will be able to reason them out readily. The introductory portion may even include fractional division. Example: $\frac{1}{2}$ of $10+3+1+10-2-10$, etc. This exercise also may be done orally.

As I have experienced it, the use of a number runner develops interest and pleasure in number exercises, as well as a mastery of number concept in all pupils, regardless of their number I.Q. at the outset of the term. We play at numbers all year. The writing of number figures and number problems is an easy step after mental development of visual number sense. I use the workbooks as a secondary phase in teaching numbers in the first grade. A wall runner and a set of ten cardboard squares in an envelope for each child, to show the number facts in addition and subtraction on his desk, are sufficient equipment to develop amazing arithmetic skill in the first grade. Of course, this does not eliminate the measuring and other phases of number work, which take other equipment.

In his article "You Asked About Arithmetic" in the October, 1959 *Grade Teacher*, Dr. Ben A. Sueltz states: "There is no exact agreement as to how much arithmetic we should have in Grade One, but we do agree that there should be a considerable amount, and that it should not be rote memorization. It should be an arithmetic of thinking, discovery, and learning." It appears to me, that the number runner is an ideal device for carrying out Dr. Sueltz's suggestion.

The principal values of the number runner in the intermediate grades, where it may be displayed in a suitable place above manual reach, are threefold: (1) It provides a ready number reference for a quick glance by pupils while they are working arithmetic problems at their desks, much as a clock or calendar are referred to in checking the time of the day or month. (2) It is an aid and a timesaver for the slow learner in arithmetic who has not mastered number facts automatically and who must continue to reason them out visually. In other words, the runner serves as a number dictionary of "glance access," lending support to the insecure and slow pupil, who must think through number processes and outcomes. (3) It serves as an excellent ready device for utilization during supervised arithmetic instruction in new number developments.

The peg board—a useful aid in teaching mathematics

ALAN A. FISHER *Mt. Tabor School, Portland, Oregon*

A peg board* is an excellent arithmetic teaching aid. It will assist the teacher in presenting new concepts and help the students visualize these ideas in a concrete form. It is useful at any elementary level and is helpful in making algebra and geometry understandable. (Fig. 1)

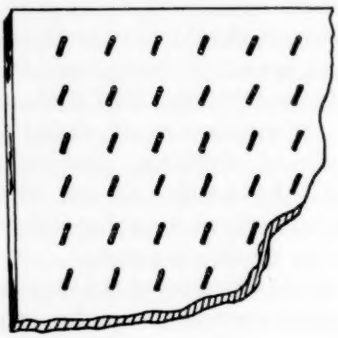

Figure 1

How to use it

Perimeter and area

Colored rubber bands slipped over the pegs make an attractive teaching aid in presenting areas and perimeters of rectangles, squares, parallelograms and triangles. An arrangement like Figure 2 can lead to discussions, such as: "What a square looks like," "Can any smaller squares be made by using more rubber bands and the same four pegs?" (Fig. 3) "How many triangles can be formed?" "What a right triangle is and how to determine its area." The perimeter may be measured with a dressmaker's cloth tape and the area determined in square inches. Bisecting angles look similar to Figure 4. The distances AB, BC, CD, and AD are

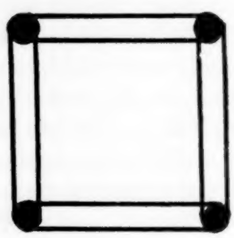

Figure 2

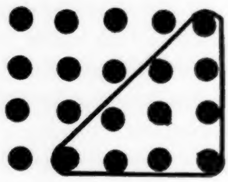

Figure 3

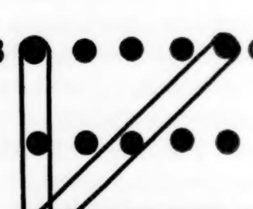

Figure 4

* The peg board may be made by gluing wood dowels cut into one-inch lengths in the holes of the Masonite peg board, leaving two-inch spaces between pegs. The material can be purchased at any building-supply store or lumberyard. The size of the board may vary, but a 2 by 3 foot size with 144 pegs has proved satisfactory for me. Most 100 boards have 10 rows with 10 pegs each but many times a larger number of pegs are needed, and those pegs not in use can be separated by rubber bands. A white background sets off any display which is arranged on the pegs, so that the display can be seen from any part of the classroom.

equal, which shows that the figure is a square. It may also be observed that the diagonals bisect the angles.

Place value and hundred board

The board can be used in developing the idea of grouping by tens and place value. One-inch wooden beads or blocks with holes large enough to slip over the pegs will help with the explanation. Start with the basic idea of how many blocks can be put on the first row, giving the idea of ten. If two rows are completed, we have twenty, etc. This material can also be used to represent parts of one hundred, to present per cents greater than one hundred, and also amounts less than 1 per cent, if washers are slipped over one peg.

Commutative law for multiplication

The commutative law for multiplication can be demonstrated by placing the blocks over the pegs in this design.

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· · · ·
· · · ·

After this is done, ask the students if the answer will be the same by multiplying three times four or rotating the board a quarter turn and showing four times three.

· · ·
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· · ·

The law of compensation, or the idea of inverse variation, can be demonstrated very effectively, too. Start the discussion by using the chalkboard to present $6\times4=24$, $(6\div2)\times(2\times4)=24$, or $(2\times6)\times(4\div2)=24$. Using any of the combinations, the answer will be 24. Then demonstrate on the peg board with the aid of rubber bands. One may present the problem, "How could we change the dimensions of a fence and still have the same area?" (Fig. 5)

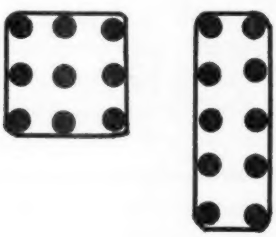


Figure 5

Squares of numbers

To represent squares of numbers, place beads on the pegs to form squares, four beads thus

· ·
· ·

will represent 2^2; nine beads thus

· · ·
· · ·
· · ·

will represent 3^2. This can continue, being limited only by the size of the board, possibly 10^2 or 12^2.

Equal ratios

Equal ratios are made more interesting by using beads to show all of one ratio and only half of the proportionate ratio on the adjoining pegs. Students can complete the ratio. See Figure 6.

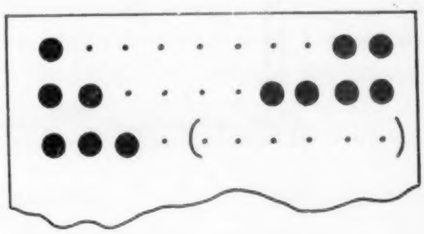


Figure 6

Co-ordinate axes

In the selection of quadrants in teaching graphing with co-ordinate axes, it is helpful to start with a number line locating the positive and negative values on the hori-

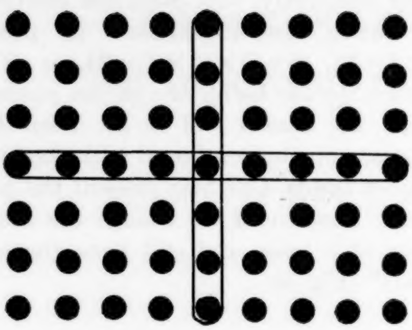


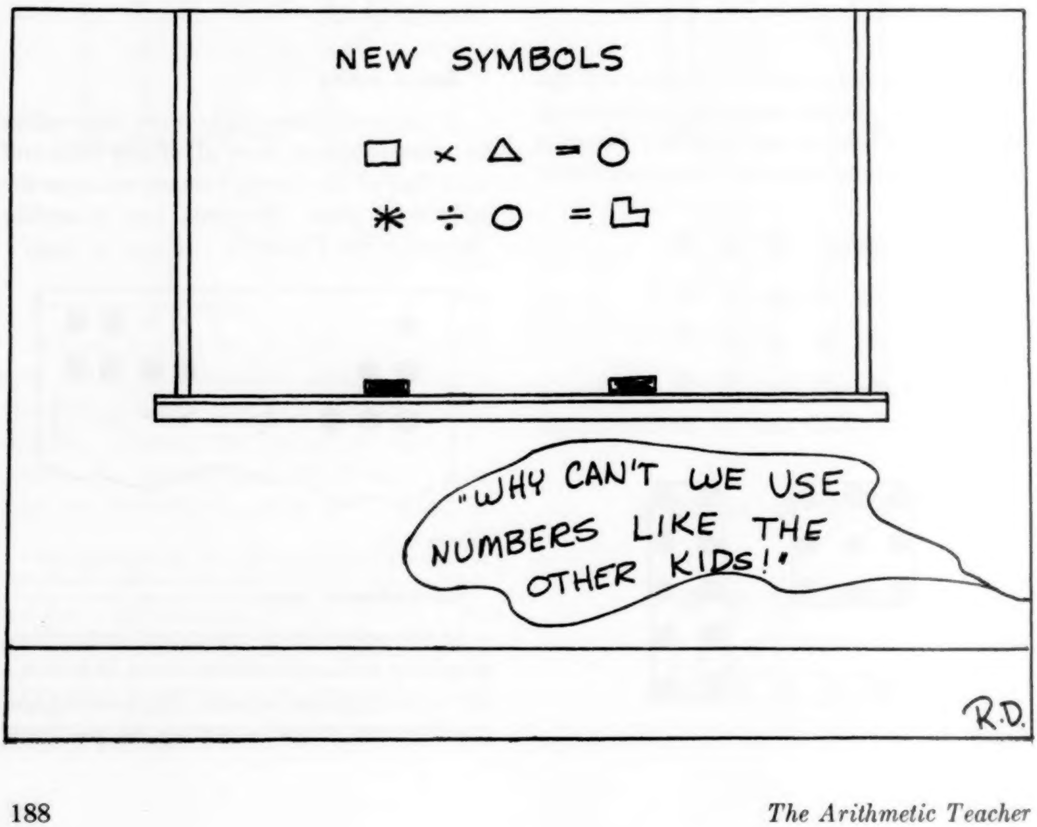
Figure 7

zontal axis. This fixes in the student's mind the idea as to which direction to count for the value of x, or the first of the ordered pair. Then, work with the vertical axis may be begun in order to give the students practice in placing the value of the second of the ordered pair, or y coordinate. When graphing, the position of the rubber bands can be adjusted to suit the demonstration. (Fig. 7)

Degrees in a polygon

The formula $180° \ (n-2)$ is used in determining the total number of degrees of the interior angles in any polygon and can be made clear by starting with the concept of 180° in a triangle. Students will enjoy forming as many triangles as possible by stretching rubber bands from *one point* to all other points or pegs in the polygon's perimeter. Interest will be high when they discover that the number of triangles formed is always two less than the number of sides of the polygon.

As teachers use the peg board, they undoubtedly will see many other uses for this device in representing mathematical ideas.



Arithmetic in science and social studies

Dr. W. A. Brownell, speaking at the Conference on Elementary School Mathematics sponsored by the School Mathematics Study Group held in Chicago, 1959, suggested that "We can go too far if concern for the mathematics of arithmetic leads us to disregard other considerations in working out the program. Whatever the mathematical content selected and whatever the sequence of learning experiences decided upon, we must be sure that we do not exceed the ability of children to learn with understanding and with profit to themselves; nor can we afford to forget that the ultimate purpose of arithmetic in the schools is not solely the preparation of mathematicians (though more and better mathematicians should be produced), but more efficient, more intelligent, and richer and happier lives for all who are subjected to arithmetic instruction." Arithmetic can make a contribution toward achieving this ultimate purpose by increasing the children's understanding of science and social studies. The development of many important concepts in these two fundamental areas of the elementary school is dependent upon understandings gained in arithmetic.

The two science activities which follow were developed by the Elementary School Science-Math Project Committee, Arlington County Public Schools, during 1959–60. Acknowledgement is made to Mrs. Jean Sheldon, Helping Teacher for Science, Arlington County Schools, and to the following committee members:

Tuckahoe School Staff

Miss Gertrude Smith, Principal
Mrs. Kathleen Marshall, Grade 4
Mrs. Dorothy Arnold, Grade 6
Mrs. Irene Sarris, Grade 2
Mrs. Helen Marmarosh, Grade 2
Mrs. Joan Deardorff, Grade 2

Science Advisory Committee

Francis J. Heydon, S. J.
Mr. Waldo E. Smith
Mr. Alexander B. Costea, Jr., TV Science Teacher for the Greater Washington Educational Television Association.
Professor Ellis Haworth
Mr. Harry Polacheck

Measuring evaporation of water, a primary-grade activity

Concepts

Evaporation occurs more rapidly at some times than at others. A finger measure can be used to measure the amount of evaporation day by day. The amount of evaporation may be found by subtracting.

Activities

1. Fill 4 to 6 glasses with water.
2. Observe daily over a two-week period.
3. Measure the loss of water day by day and compare the total loss with the original amount.
4. Determine loss by subtraction.
   a. Measure by fingers the amount of water in each jar from bottom up.

Table 1
Weights

Object	*Dry weight*	*Wet weight*	*Loss of weight in water*	*Specific gravity*
rock	50 gr.	30 gr.	50 g. − 30 g. = 20 g.	$\frac{50\text{ g.}}{20\text{ g.}} = 2.5$

(Loss is measured by applying finger-wide measure. Use index finger each time.)

b. Keep daily record.

Mon.	*Tues.*	*Wed.*	*Thurs.*	*Fri.*
12	11	9	8	8
−1	−2	−1	−0	−1
11	9	8	8	7

c. Make all measurements at the same time each day and to the nearest whole number.

d. Help children see the relationship between the kind of weather and the amount of evaporation.

Determining specific gravity, an intermediate-grade activity

Concepts

Specific gravity is the number of times a substance is as heavy as an equal volume of water. The loss of weight of a solid immersed in water is equal to the weight of a volume of water equal to the volume of a solid being weighed.

Activities

A. Help children build the above concepts by carrying out the following demonstration:
   1. Weigh a rock.
   2. Immerse the rock in a *full* glass of water and catch the water that overflows.
   3. Weigh the "overflow."

B. Then:
   1. Immerse the rock in water, weighing it with a spring scale.
   2. Weigh the rock in air.
   3. Subtract the weight in water from the weight in air. The result obtained should equal the weight of the "overflow" in A-3 above.

C. Make a table of dry weights, wet weights, and specific gravity for each object (see Table 1).
   1. Record name of object.
   2. Weigh the object in air. Record dry weight.
   3. Immerse object in water. Record wet weight.
   4. Determine loss in weight and record.
   5. Divide $\frac{\text{dry weight}}{\text{loss of weight in water}}$ to find specific gravity.

Large numbers in social studies

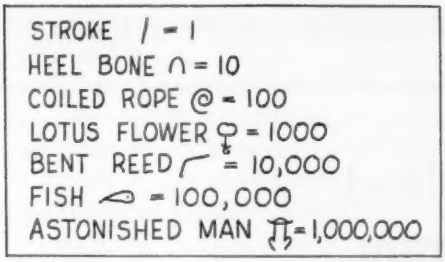
Thanks to Miss Connie Brucken, fifth-grade teacher, Highland Park School, Fort Dodge, Iowa, and to the consultant for the Rapid Learner Program, Miss Gladys Grimjes, for the following experience with using large numbers in social studies:

> This was a lesson in correlating numbers and social studies. Each child takes the *Young Citizen* and *News Times,* weekly publications for elementary grades. The study of large numbers led into the study and making of graphs and map locations of the ten largest cities in the United States. This interesting topic made an attractive bulletin board.

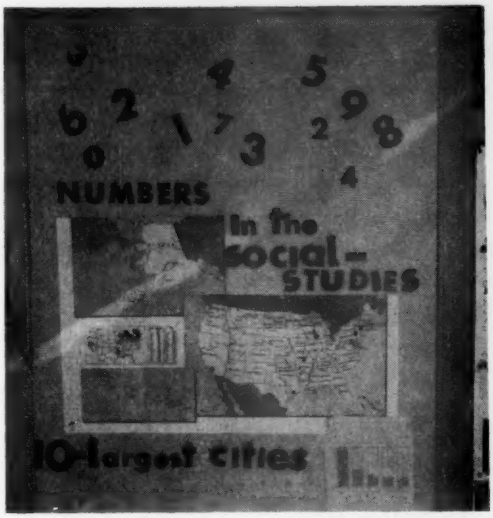
Egyptian numerals

As children study Egypt they will find it interesting to learn about the number system used by the early Egyptians. Activities may include:

1. Writing names of numbers using Egyptian symbols



2. Reading names of numbers written by other children
3. Analyzing differences between our Hindu-Arabic system of numeration and the Egyptian numeration system.



Using data from social studies

Tabulated social studies data can be used to solve problems which will help children appreciate concepts of sizes, distances, heights, etc. For example, determining difference between pairs of numbers in Table 2 will help children appreciate how the Panama Canal shortened the travel distance from one city to another. As children use source materials, there are many opportunities for them to organize and arrange data in order to clarify ideas.

Table 2

From	*To*	*Miles around South America*	*Miles through Panama Canal*
New York	San Francisco	15,348	6,059
Honolulu	New Orleans	15,942	7,051
Seattle	Liverpool	16,806	10,014
New York	Manila	19,530	11,585
New Orleans	San Francisco	13,644	4,698

For instance, they learn that if all the people in the world were evenly divided there would be about 42 people per square mile. Further study reveals vast differences in the number of people per square mile in different parts of the world. Alaska has an average of only one person for each 10 square miles, while Java has an average of about 1,000 people per square mile. Next, they find data on the number of square miles of the continents and the total number of people for each continent. By arranging these data in tabular form (see Table 3), they can estimate the average number of people per square mile and then compute to find the average number per square mile. Study of the results will give children an appreciation of the differences. Children might follow up such a study by determining like differences for cities or states within our own country.

Table 3

Continent	*No. of sq. miles*	*Population*	*Average no. people per sq. mi.*
Asia	16,793,000	1,314,000,000	?
Europe	3,762,000	533,000,000	?
North America	9,375,000	213,000,000	?
Africa	11,600,000	198,000,000	?
South America	6,846,000	108,000,000	?
Australia	2,975,000	8,250,000	?

The Greater Cleveland Mathematics Program

B. H. GUNDLACH
Bowling Green State University, Bowling Green, Ohio

Editor's note

There is much current interest in The Greater Cleveland Mathematics Program (G.C.M.P.). This interest relates to the mathematical content of the Program, to its plan of in-service education, and to its organizational structure, procedures and financial support. Readers of THE ARITHMETIC TEACHER will welcome this G.C.M.P. report from Professor Gundlach, Chief Consultant to the Program. (J.F.W.)

The G.C.M.P. attempts to lead its students to the challenging and exciting phases of mathematics-in-the-making by having them *rediscover* the fundamental patterns and by having them *re-create* the basic symbolism, thus making the learning of mathematics to a large extent self-motivating. Problem situations are presented to students *as if* they had not been explored already by the great geniuses of the past and present. These situations are presented in such a manner that pattern discovery has a good chance of taking place almost spontaneously; then students are guided to a point where the established symbolism for the rediscovered pattern seems almost a necessity. *It is hoped that* students will come away with the feeling that they, for themselves, have seen the relations, invented the symbols, put them together in meaningful sentences, and, in general, have had a genuine part in making this or that piece of mathematics.

This takes wise guidance by the teacher. Such guidance requires that the teacher know the piece of mathematics thoroughly from beginning to end, and more, that he or she have a fair appreciation of how and where a particular piece fits into the large, continuous whole which is mathematics.

A most serious problem was, and still is, to provide a sufficient number of teachers with the background necessary to create such challenging and exciting classroom situations. In addition to being a good teacher, this requires substantial subject-matter mastery, considerably more, in most cases, than at least most elementary teachers are given during their college preparation. Thus the first task of the G.C.M.P. has been to establish a large, well-organized in-service teacher-training program.

The G.C.M.P. In-Service Program

Three chief instructional means and a number of minor assists were used to implement the in-service program:

1. Regularly scheduled lectures by professional mathematicians and mathematics educators
2. Printed materials with which to prepare for and to follow the lectures, supplemented from time to time by sets of

classroom suggestions and "enrichment materials"

3. A series of instructional TV programs (a daily half-hour) offered as a public service by Cleveland's KYW-TV, Channel 3, in co-operation with the Educational Research Council

Among the many auxiliary instruction measures were: classroom demonstrations with children; slide tape programs on special topics and approaches; and public-relation presentations to all the P.T.A.'s of the participating schools.

The lectures are designed for one or two grade-levels at a time. They are held in several centers in the suburban districts of Cleveland, so that no teacher will have to travel an unduly large distance to reach them. The recognition given the teachers for regular attendance varies from one school district to another. Released time and/or credit points on the salary scale of the system are commonly used.

The printed materials go through three general stages:

1. In the first year they consist merely of lithographed lecture notes and classroom suggestions.
2. In the second year they are transformed into teachers' guides and sets of pupils' exercises, prepared and written by teams of writers selected from the program's large number of enthusiastic teachers. These writing teams are guided by the council's experts, and the considerable consultant resources of the council are available to them.
3. In the third year they should be ready as a permanent, hard-cover, textbook series.

The over-all program is organized with a staggered structure, calculated to expand gradually, over a period of 5 or 6 years, from kindergarten through the 12th grade.

During the initial training period, while one group of teachers is being instructed for the first time, no immediate classroom implementation is advocated. Only after the instruction has reached a relative conclusion for that particular level, and after the teachers have become well-acquainted with materials and approaches, are they encouraged to present these materials to their students and to make use of the "new" approaches. In general, the first semester of each school year is given to the in-service program, while the second semester is dedicated primarily to classroom implementation.

For the implementation phase at every level, the program provides a number of specially trained fieldmen who are on call for any emergency that might arise in any of the classrooms.

The TV programs serve as an additional means to help with the implementation of materials and teaching approaches by suggesting definite teaching procedures and presentations of materials.

The in-service program as a whole is being critically examined through a continuous process of evaluation that is being carried out in the various classrooms of the participating schools. This process helps to direct the over-all program in such a manner that weak points can be spotted rapidly and remedial measures or changes be implemented with a minimum of delay.

The mathematical content

Arithmetic is treated as a fundamental part of mathematics, not as a mere collection of "facts," "algorisms," and "formulae." Since arithmetic *is* a portion, and a very essential one at that, of mathematics, it is deeply concerned with "making proof." For example, why does $3+5=5+3$, while $3-5\neq5-3$? What is there in the very depth, at the very beginning of our number system, that ascribes this important property to addition but withholds it from subtraction? A study of commutativity is undertaken. Again, why is 7×17 equal to 119? Why does the com-

mon stereotype algorism of written multiplication work? In examining it closely and carefully, it is found that it is regulated by the *distributive* and *associative* properties of multiplication and addition together with the basic concept known as place value. Does the symbol "123" always mean "one hundred twenty-three"? If it does, why does it? If it does not, how would its value be determined in every case? Answers to these and many related questions are discovered in a study of *numeration systems* where it is learned that our ordinary base-ten arithmetic is neither the only possible nor is it necessarily the most convenient arithmetic. Children learn to select the most suitable arithmetic for any given purpose and, at the same time, gain unsuspected insights into the inner workings of our own arithmetic, which most of us take for granted.

Why, for example, is the sum of two odd numbers always even, while their product is always odd? What does "always" mean in such statements? We find a surprisingly simple answer in *proof*. Even within the narrow confines of our familiar base-ten arithmetic, strange and bewildering situations arise from time to time. In the classic approach we avoided such problems or treated them as nonexistent or as "forbidden." In the modern approach they are sought out and solved. Is it true, for example, that the sum of five and seven could be ten instead of twelve? It is true. For example, five children in this class wear pull-overs, and seven children wear glasses. Yet when all the children who wear glasses and/or pull-overs are asked to stand, only ten children rise. How could that be? And what does this "and/or" stand for?

The answer to this and numerous similar questions lies in the idea of *sets*, introduced in extremely simple and pictorial language.

Set language helps in several other ways. One of our greatest preoccupations in elementary arithmetic is teaching the children a sound concept of number. What is number? Is "3" a number? Is "III" also a number? And where does the *word* "three" come in? To clarify situations of this nature, we study a few very simple sets. Set language helps small children to distinguish clearly between *numerals* and *numbers,* a distinction without which arithmetic would soon become hopelessly confusing.

In all of these considerations of arithmetical nature there is concern for *form* and *value.* Any arithmetical value (or number) can have many different forms. The number 5, for example, could be expressed as $2+3$, or $9-4$, or $\frac{1}{2}\times 10$, or $20-4$, or $\sqrt{25}$, and in many other forms all having the same value 5.

In the traditional approach the concept of value and its familiar "How many?" question have been emphasized almost to the complete exclusion of the equally important concept of form. In this program an effort is made to strike a sound balance between these two basic concepts of mathematics by learning to make changes of form (transformations) when the values remain equal (equivalence), and by becoming concerned with changes of values when the forms remain unchanged. This much attention is given to the basic properties of form and structure.

As the term "form" implies, with the form concept we stand on the threshold of creative activity in arithmetic. To present one very small example: Have you ever considered the many different forms, all made with *four* 3's (no more and no less), in which one can write, for example, the first ten natural numbers? You will ask what the purpose of this sort of work is. Simple. Why not try, for example, to have children write all natural numbers from 1 to 100 with four 3's, all natural numbers from 11 to 100 with four 3's! It is fun; it is also good mathematics. It is this combination of ingredients which counts. New symbolic forms and new operations can be introduced easily in this manner and will provide the children

with a real opportunity to exercise their creative imagination.

The number	*Forms used*
$1 = \frac{33}{33}$	place value notation and division
$2 = \frac{3}{3} + \frac{3}{3}$	division and addition
$3 = 3 \times 3 - (3+3)$	multiplication, subtraction, addition, parentheses

Children learn that any genuine piece of arithmetic is constructed from three basic components:

1. Numerals (like 0, 1, 2, 3, · · ·)
2. Operations (like $+$, $-$, $\times$, $\div$, · · ·)
3. Relations (like $=$, $\neq$, $<$, $>$, · · ·)

They also learn early to write their verbal questions and statements in acceptable mathematical form, using "placeholder equations" or "open sentences."

Writing equations as translations of verbal statements into the language of mathematics helps students with the solution of word problems. They now can attack such problems in a much more systematic manner than was hitherto possible.

The form concepts of elementary mathematics likewise find a strong expression in the approach used to elementary construction geometry (straightedge and compass). This is initiated late in the first grade and carried through in continuous fashion to the more sophisticated level of formal geometry. Again, the language of sets is extremely helpful in building sound geometric concepts on the levels of informal nonmetric geometry.

Observations

Through tasks such as those and many others the children strongly experience the gradual emergence of elementary mathematics as a well-structured system, in which everything has its meaningful place and is related to the whole in a well-determined manner. It is surprising to see how comfortably children can be made to feel at home in such a system.

Discovery, challenge, and use of creative imagination are the true hallmarks of the G.C.M.P. approach. Having created this attitude of open-minded and joyful acceptance of mathematics, there seems almost nothing that children are not willing and able to learn.

"It is hardly necessary to say that the old expressions, 'borrow' and 'carry,' in subtraction and addition are rapidly going out of use; they were necessary in the old days of arbitrary rules, but they have no advocates of any prominence to-day."—D. E. Smith, *The Teaching of Elementary Mathematics*, 1901.

Books and materials

The Story of Mathematics: Geometry for the Young Scientist, Hy Ruchlis and Jack Engelhardt. Irvington-on-Hudson: Harvey House, 1958. Cloth, 148 pp., $2.95.

Plane and solid geometry may provide a fascinating study of designs and patterns. However, many persons who study these subjects are not aware of the extensive use of designs in nature, or the many industrial uses of the mathematics of form. The usual method is that of teaching for transfer to the areas of natural science and industry *after* the subjects themselves have been studied. A second approach is that of pointing out the many practical uses, *then* teaching them as subject matter. It is this second approach which has been used in *The Story of Mathematics.* The reader is led to feel the tremendous impact of the mathematics of form in the world around him; then the mathematics of spheres, rectangular solids, cylinders, lines, angles, circles, and polygons is presented in a very interesting manner with many excellent illustrations.

The primary emphasis is upon planes and solids; however, some attention is given to problem-solving by the use of both arithmetic and algebra. It is pointed out that mathematics is a very useful tool in solving unusual and difficult, yet practical, problems in many areas, including carpentry, navigation, and space travel. Brief emphasis is placed upon mathematical games and puzzles, and four rather interesting problems and their solutions serve to illustrate the practical use of mathematics. A short list of other books which present a similar approach to mathematics is given.

Both the style of presentation and the vocabulary make this book somewhat beyond the level of the elementary-school child. Children likely would enjoy the outstanding illustrations, but most of the reading would be difficult for the average child of twelve years of age. The most effective use of the book with elementary-school pupils would include the close guidance of an adult. Pupils of junior-high-school level and above will gain much from individual use of the book, and it should prove most beneficial for talented pupils at all levels. It provides teachers with the unusual resource of many attractive and useful designs and patterns, which are an integral part of plane and solid geometry.

Teachers and pupils need to strengthen their understandings and appreciations of the many practical aspects of mathematics. *The Story of Mathematics* is an excellent resource for moving toward this goal.

LONIE RUDD
Tufts University
Medford, Massachusetts

More about 1960-1961 committees

The following omissions were made in giving the list of names of persons appointed to NCTM 1960–1961 Committees and Representatives (February, 1961 issue). We wish to correct our mistakes.

Executive Committee

Phillip S. Jones, ex officio, Ann Arbor, Mich., Chairman
Myrl H. Ahrendt, ex officio, Washington, D.C.

Editorial Committee, 27th Yearbook (Talented)

Vincent J. Glennon, Syracuse, N.Y.

Membership Committee

Mary C. Rogers, Westfield, N.J., Chairman
Pearl Bond, Beaumont, Tex.
Janet Height, Wakefield, Mass.
Harold J. Hunt, Seattle, Wash.
Faith Novinger, Washington, D.C.
Lucille Houston, Racine, Wis.
Florence Ingham, Bartlesville, Okla.

ROCM Project

Frank Allen, LaGrange, Ill., Director

Kindly regard these as a part of the report appearing in the February issue of THE ARITHMETIC TEACHER.

E. Glenadine Gibb
EDITOR

Steering Committee

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E. G. Begle, New Haven, Conn.
Kenneth Brown, Washington, D.C.
Edwin C. Douglas, Watertown, Conn.
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Mildred Keiffer, Cincinnati, Ohio

Professional dates

The information below gives the name, date, and place of meeting with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE ARITHMETIC TEACHER. Announcements for this column should be sent at least ten weeks early to the Executive Secretary, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C.

NCTM convention dates

Joint Meeting with NEA
June 28, 1961
Atlantic City, New Jersey
M. H. Ahrendt, 1201 Sixteenth Street, N.W., Washington 6, D.C.

(*For other meeting dates, see page* 198.)

Twenty-first Summer Meeting

August 21–23, 1961
University of Toronto, Toronto, Canada
Father John C. Egsgard, C.S.B., St. Michael's College School, 1515 Bathurst Street, Toronto 10, Canada

Fortieth Annual Meeting

April 15–18, 1962
Jack Tar Hotel, San Francisco, California
Kenneth C. Skeen, 3355 Cowell Road, Concord, California

Other professional dates

Illinois Council of Teachers of Mathematics

April 22, 1961
Illinois State Normal University, Normal, Illinois
April 29, 1961
Sterling Township High School, Sterling, Illinois
T. E. Rine, Illinois State Normal University, Normal, Illinois

The Greater Cleveland Council of Teachers of Mathematics

April 18, 1961
Bedford High School, Bedford, Ohio
Bessie Kisner, Strongsville High School, Strongsville, Ohio

Men's Mathematics Club of Chicago and Metropolitan Area

April 21, 1961
YMCA Hotel, 826 South Wabash Avenue, Chicago, Illinois
Vernon R. Kent, 1510 South Sixth Avenue, Maywood, Illinois

California Mathematics Council

April 28–20, 1961
Santa Monica, California
Miss Mary Lou Hood, 3414 North Eckart, South San Gabriel, California

Pennsylvania Council of Teachers of Mathematics

April 29, 1961
Clarion State College, Clarion, Pennsylvania
Earl F. Myers, 322 13th Avenue, New Brighton, Pennsylvania

Association of Mathematics Teachers of New York State

May 5–6, 1961
Hotel Syracuse, Syracuse, New York
Edward E. Sherley, Mt. Pleasant High School, Schenectady 3, New York

Michigan Council of Teachers of Mathematics

May 5–7, 1961
M. E. S. Camp, Saint Mary's Lake, Battle Creek, Michigan
Mrs. Marjorie Pickering, 5198 Coldspring, Birmingham, Michigan

Chicago Elementary Teachers Mathematics Club

May 8, 1961
Toffenetti's Restaurant, 65 West Monroe Street, Chicago, Illinois
Mildred C. Rogers, Warren Elementary School, 9210 South Chappel Avenue, Chicago 17

Men's Mathematics Club of Chicago and Metropolitan Area

May 19, 1961
YMCA Hotel, 826 S. Wabash Ave., Chicago
Vernon R. Kent, 1510 South Sixth Avenue, Maywood, Illinois

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EDMUND INSTRUCTIVES

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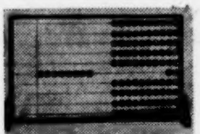
THE BASE-TEN DEMONSTRATION ABACUS



Its arrangement makes this abacus easy for the teacher to demonstrate and for the child to understand what "really happens" at each step of the arithmetic process being presented. Whether it is a simple presentation of number groupings in the first grade, an explanation of our system of place value, addition and multiplication with carrying over, decomposition in subtraction, or division with remainders—the carefully planned set up of the Base-Ten Teaching Abacus makes it visually clear just what the operation involves with tangible manipulative materials. This, in turn, prepares the child for a true understanding of the processes involved when he advances to the use of the abstractions that we call numbers.

The Base-Ten Teaching Abacus is a large (19" high x 24" long) frame with ten horizontal rods, designed to stand upright on teacher's desk or table. On one side of the rods are the unit counters, ten to a rod, one hundred in all. On the other side of a vertical dividing rod are the ten-value counters, nine of them, representing ten-values from the 'teens up to the nineties. The counters are large and the frame itself is big and roomy enough so that any demonstration the teacher makes on it is clearly visible and understandable to the student at the back of the room.

In learning to group and manipulate the counters on an abacus the class establishes an understanding of arithmetic that goes beyond rote learning.

Complete instruction for use, with examples, come with the Base-Ten Teaching Abacus.

Stock No. 80,067-SJ ....$13.50 Postpaid

SPACE RINGS . . . The Floating Mobile!

Kit consists of 4 perforated aluminum rings—4" to 8" diam.; metallic colored yarn; hoop; hanger; directions.

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ABACUS

Our abacus is just the thing to use in class to teach place value and number concepts. It is more effective than a place frame. It is of our own design and is 9¾" wide and 7¾" long. It is made of a beautiful walnut wood, with 6 rows of 10 counters. Complete instructions are included with every abacus.

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ABACUS KIT—MAKE YOUR OWN!

Making your own Abacus is a wonderful project for any math class, or math club, or as an enrichment unit. Our kit gives you 60 counters, directions for making your own Abacus and directions for using our Abacus.—Makes one Abacus.

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For 1,000 counters and one set of directions, makes 16 Abacuses—
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Stock No. 50,234-SJ—100 Brass Rods ....$6.90 Postpaid
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Why do we have differences in time occurring simultaneously at different points around the world? Why must we have an international date line to adjust these differences? Teachers who have found their classes have difficulty understanding the answers to these questions will discover that the WORLD-TIME INSTRUCTIVE is a valuable teaching device. The logic of world time is made visual and lucid even to the slower student when a demonstration is given with this new instructive. The impact is much greater than when only a globe is used for the explanation. Printed on durable laminated sheets that can be easily wiped clean.

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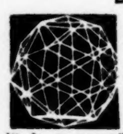
Stock No. 70,278-SJ ....$2.95 Postpaid

TEACHING AIDS

D-STIX CONSTRUCTION KITS



D-Stix clarify geometric figures for the young by actually demonstrating them in three dimensions. They make recognition and understanding fun. They can also be used for showing molecular structures in science classes, concepts of elementary science and physics. These modern construction kits are far superior to older style wood or metal construction kits.

220 pieces, 5, 6 and 8 sleeve connectors, 2", 3", 4", 5", 6" and 8" colored D-Stix—
Stock No. 70,209-SJ ....$3.00 Postpaid
370 pieces, incl. 5, 6, and 8 sleeve connectors, 2", 3", 4", 5", 6", 8", 10" and 12" D-Stix in colors—
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452 pieces, includes all items in 70,210 above plus long unpainted D-Stix for use in making your own designs—
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